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THE MATHEMATICS TEACHER

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A Study of Certain Mathematical Abilities in High School Physics*

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I. THE ORIGIN OF THE PROBLEM

Introduction

THE PROBLEM of the functioning of the mathematical training of the student in high school physics has received much attention during the past decade. This attention has ranged from magazine articles advocating the *dematheniatizing* of high school physics to carefully planned studies to determine the kind and amount of mathematics needed in the course. There has been much difference of opinion as to whether or not too much emphasis has been placed upon the mathematical aspects of the subject. The earlier discussions were much concerned with this phase of the problem, but not much was done to settle the question on an objective basis. Hughes has summarized the arguments on both sides of this question and has presented certain conclusions regarding this controversy.[1]

* This is a part of a more complete study by Mr. Carter. The remainder of the study will appear in later issues of THE MATHEMATICS TEACHER.—THE EDITOR.

[1] Hughes, J. M. "Shall We Mathematize or Demathematize High School Physics?" *School Science and Mathematics*, Vol. XXIV, pp. 916-921

Even the best of the earlier studies of the mathematical equipment of physics students were chiefly concerned with the use of facts and test data in support of arguments for or against certain types of curricula in physics or in support or criticisms of current practices in the teaching of physics or mathematics.

The Western Reserve Study is typical of the earlier investigations. As a part of this study a test made up of fourteen physics problems involving simple computational abilities was administered to 238 physics students in a large high school. In an article based upon the results from this test, Randall, Chapman, and Sutton say:

> . . . Taking the whole body of pupils, on the average each problem is successfully solved by fifteen per cent of the pupils; no problem is solved by more than sixty-nine per cent, while two straightforward problems were out of the range of every pupil. This reveals a condition truly surprising. The extent of the lack of comprehension shown therein of the numerical relations of the simplest and most fundamental principles of physics is certainly startling, and lends support to the criticism often heard that the average high school student of physics acquires merely a mass of disconnected facts, with but little notion of the underlying and unifying principles. It is clear that the present method of teaching physics fails in so far as it attempts to give the pupil any knowledge of the principles which lie back of the common numerical problems.[2]

In a similar study Neville, after discussing the poor performances of his chemistry students on a simple mathematics test, says:

> Their errors were not mechanical but were in reasoning. May this not be due to emphasis in their training on the mechanical processes involved in problems rather than upon the meaning? . . . Students come up without a clear conception as to whether a meter more nearly approximates a mile, a yard or an inch.[3]

These criticisms are representative of the many that were made when science teachers began to realize that the mathematical preparation of their students could not be taken for granted.

A second and more recent method of attack on this problem has been made from the viewpoint of attempting to discover what mathematical elements are found in representative texts in high school physics, or from the viewpoint of finding the relationships between success in physics and various types of abilities, including certain

[2] Randall, D. P., Chapman, J. C., and Sutton, C. W., "The Place of Numerical Problems in High School Physics," *School Review*, Vol. XXVI, p. 41.

[3] Neville, H. A., "Mathematics and Science," *Mathematics Teacher*, Vol. XX, pp. 22-23.

rather general mathematical abilities. Practically all of the more carefully planned studies of this type have apparently been made under the assumption that the mathematical abilities involved in high school physics are computational in nature, and that the mathematical difficulties of high school physics students are found in the manipulation of mathematical symbols or in the functioning of some general power such as reasoning ability or problem solving ability in the actual solution of physics problems. The more recent methods of approach to this problem may be illustrated by references to the studies in this field.

Previous Studies of Mathematical Abilities in High School Physics

In one of the best of the more recent studies, Kilzer attempted to find out what elements of mathematics are useful in high school physics.[4] By means of a questionnaire he ascertained the names of the five most frequently used physics textbooks in 345 high schools of Iowa in the fall of 1927. He then solved all of the problems given in these five textbooks and their manuals and recorded the processes used. Upon the basis of the data thus obtained, he constructed an inventory test for the mathematics needed in high school physics.[5] Among Kilzer's conclusions the following are pertinent to this study:

1. The five physics textbooks most frequently used during 1927-1928 in the 345 high schools included in this study are, in descending order of frequency of use: Fuller, Brownlee, and Baker; Millikan and Gale (and Pyle); Carhart and Chute; Black and Davis; and Dull.
2. Physics was required in approximately seventy per cent of the 345 Iowa high schools included in this study, and is taught in grade twelve, only, in approximately eighty-four per cent of these schools.
3. The mathematics needed in solving the problems of high school physics involves a considerable body of information usually taught in arithmetic, algebra, and plane geometry. Not much trigonometry is needed.
4. Most of the mathematics needed to solve high school physics problems is not very difficult.
5. Most pupils who are eligible to take physics use their mathematics poorly. The median score on this test is only slightly above half of a perfect score on the test as a whole.

[4] Kilzer, L. R., *The Mathematics Needed in High School Physics.* Unpublished Ph.D. thesis, University of Iowa, 1928.

[5] Kilzer, L. R., and Kirby, T. J., "An Inventory Test for the Mathematics Needed in High School Physics," Public School Publishing Company, Bloomington, Ill., 1929.

6. There is a definite need for maintenance drills covering the mathematical items and processes needed in high school physics. A work-book providing adequate drill on these items and processes would be valuable.

7. There is a considerable difference in the preparation of different pupils on certain items and processes.[6]

Kilzer's study contains a fairly specific analysis of one group of mathematical abilities involved in high school physics, and is an important contribution to the field of the psychological analysis of subject matter on the high school level. His study, however, is limited to the mathematical abilities or operations that are involved in the solution of problems in physics.

Reagan made a somewhat less specific study of the mathematics involved in solving high school physics problems.[7] He solved and analyzed 241 problems in Millikan and Gale's *A First Course in Physics*[8] and classified the results of his analysis under the headings of arithmetic, algebra, and geometry, with certain large subdivisions under each. For example, under arithmetic abilities his subdivisions are addition, subtraction, multiplication, division, common fractions, denominate numbers, percentage, and mensuration. No tests were constructed in connection with this study, and it was not concerned with facts, principles, or processes treated in the body of the text.

Burgess constructed a series of tests to measure various general abilities in high school physics.[9] He found the following factors, in order of importance, to be included in what he called physics ability:

1. Interest in physics
2. Simple mathematics of physics
3. Observation aptitude
4. Reading comprehension
5. Number series and logic

Gribble made a study of the relationship between success in physics, as ordinarily taught, and (1) reading ability, (2) general intelligence,

[6] Kilzer, L. R., *The Mathematics Needed in High School Physics*. Unpublished Ph.D. thesis, University of Iowa, 1928. These conclusions are quoted from a summary of the study in the November, 1929, issue of *Science Education*, pp. 343-344.

[7] Reagan, G. W., "The Mathematics Involved in Solving High School Physics Problems," *School Science and Mathematics*, Vol. XXV, pp. 292-299.

[8] Millikan, R. A., and Gale, H. G., *A First Course in Physics*, Ginn and Company, 1914.

[9] Burgess, T. O., *A Psychological Analysis of Abilities in High School Physics*, University of Iowa Studies in Education, Vol. III, No. 6, Iowa City, 1926.

and (3) certain mathematical abilities.[10] He used only ninety pupils as subjects and emphasized general rather than specific abilities. He presented the following tentative conclusions:

1. The correlation between reading ability and success in physics is low but positive.
2. The correlation between general intelligence and success in physics is low but positive.
3. The correlation between certain mathematical abilities and success in physics is higher than either of the foregoing correlations.[11]

Congdon made a study in order "to determine what training in high school mathematics is essential for success in certain college subjects that do not presuppose a knowledge of college mathematics."[12] He analyzed and tabulated the facts, concepts, skills, general processes, and methods of procedure which were used in the solutions of 572 problems in Stewart's *Physics, A Textbook for Colleges.*[13] He also made tabulations and counts of the mathematical vocabulary and the symbols employed in this text. He constructed a research test of eighteen problems involving algebraic manipulations selected from those necessary to the solutions of the problems in this text and reported the results of the administration of the test to 859 high school students who had studied algebra for at least three semesters.

Although Congdon's study is chiefly in the college field, it contains data pertinent to the present investigation. His findings, in part, are as follows:

1. Previous investigations concerning the relationship between high school mathematics and other subjects have dealt almost exclusively with facts and skills.
2. The frequency of certain words in the mathematical vocabulary determined by a careful word count of Stewart's textbook suggests the advisability of additional emphasis in high school mathematics classes upon certain concepts. Among the words and expressions which suggest important concepts in physics that might profitably receive more emphasis in mathematics are: axis, rotation, clockwise and counter-clockwise motion, positive and negative numbers, constants and variables, maximum and minimum, rates and speeds, and the series of words connected with variation and proportionality, direct and inverse."[14]

[10] Gribble, S. C., *An Investigation of Some Factors Influencing Success in the Study of Physics,* University of Iowa Master's thesis, 1924.

[11] Gribble, S. C., *Op. cit.,* p. 97.

[12] Congdon, A. R., *Training in High School Mathematics Essential for Success in Certain College Subjects, With Special Reference to Physics.* Teachers College Contributions to Education, No. 403, 1930.

[13] Stewart, O. M., *Physics, A Textbook for Colleges.* Ginn and Company, 1924.

[14] Congdon, A. R., *Op. cit.,* p. 89.

Miss Crawford analyzed four high school physics texts and determined the frequency of occurrence of mathematical terms and expressions in them for the purpose of finding what mathematics requirements should be established for the study of physics.[15] A word count was the chief technique used in this study.

Criticisms and Evaluations of the Foregoing Studies

An analysis of the procedures and conclusions in the foregoing studies seems to justify the following conclusions:

1. Many of the investigations in the field of the mathematical abilities involved in high school physics have been primarily concerned with computational abilities, the solutions of problems, or the answering of questions relating to mathematical facts and information.
2. In many studies frequent use is made of such expressions as *reasoning ability, problem-solving ability,* and other general expressions involving, perhaps, many detailed abilities which probably should be studied separately.
3. Some mention has been made of mathematical concepts in high school physics, but little seems to have been done in a detailed study of them or of their relationships to other mathematical abilities in this subject.
4. Studies involving the investigation of factors, mathematical or otherwise, influencing success in physics have dealt with fairly general abilities. In many cases the term "mathematical abilities" has been used in an undifferentiated sense with no attempt at analysis into specific components.
5. Apparently in no case has there been an investigation having to do with the student's ability to recognize the mathematical concepts which are involved in the reading of the physics text.
6. In a number of studies the conclusions have been based upon results from comparatively small groups of subjects, or upon results from research tests involving only a few test items.

It seems, then, that while a few studies have made available rather definite and valuable conclusions concerning the mathematical abilities in high school physics, many important questions have not been answered. Very little attention has been given to the kinds of

[15] Crawford, Vira M., *The Mathematics Needed as a Prerequisite for the Study of High School Physics.* Unpublished Master's thesis, Colorado State Teachers College, 1927.

mathematical abilities which are needed in reading a physics text and in understanding expositive material in physics. Attempts at determining what mathematical concepts actually occur in physics have been very limited in number and scope and have been made, for the most part, as side issues in other studies. This probably has been due to the fact that even in the field of mathematics not much has been done until recently in isolating and listing the mathematical concepts occurring in high school mathematics or in differentiating between these concepts and the mathematical facts and skills which are an outgrowth of them. There are now certain studies and techniques in the literature of high school mathematics which are related to the problems described in this chapter and which may be of value to the present study.

Previous Studies in High School Mathematics

Apparently the first attempt at formulating and validating a list of mathematical concepts was made by Raleigh Schorling during six years of research previous to 1925 in connection with his study of objectives in the teaching of junior high school mathematics.[16] In this study Schorling has listed sixty-three basic concepts and nineteen subsidiary concepts as objectives of junior high school mathematics.[17] Blackhurst makes the following comment on this study:

> The study by Raleigh Schorling is the most thorough and scientific attempt made to date to determine a specific list of objectives for junior high school mathematics.[18]

Smith and Reeve have formulated a somewhat more comprehensive list of mathematical concepts which includes most of those in Schorling's list.[19] These authors, however, do not go into detail as to the basis for choosing their list of concepts.[20]

Various other authors have mentioned certain of these concepts. Hinkle has gone to some length in presenting arguments for the in-

[16] Schorling, Raleigh, *A Tentative List of Objectives in the Teaching of Junior High School Mathematics, With Investigations for Determining Their Validity.* Published by George Wahr, Ann Arbor, Michigan, 1925.

[17] Schorling, Raleigh, *Op. cit.*, pp. 101-102.

[18] Blackhurst, J. H., *Principles and Methods of Junior High School Mathematics*, The Century Company, 1928. p. 82.

[19] Smith, D. E., and Reeve, W. D., *The Teaching of Junior High School Mathematics*, Ginn and Company, 1927. pp. 43-47.

[20] *Ibid.*, p. 3.

clusion of a number of them in junior high school mathematics.[21] Christofferson has discussed in considerable detail various ways of teaching two of these concepts, and has pointed out defects in present methods of teaching them.[22]

Summary and Conclusions

In general, there have been two methods of approach to the problem of the mathematical abilities involved in high school physics. The first has to do with a study of the mathematical equipment of the physics student with a view to adjusting the content of the course to his needs. The second is concerned with an analysis of physics textbooks and manuals for the purpose of finding what mathematical operations and abilities are involved in representative situations. It seems that the more recent and more carefully planned studies have followed the second method of attack.

The importance of various problems connected with the functioning of the mathematical equipment of the high school physics student seems to be attested by the number of studies that have been made in this field.

To one interested in certain psychological aspects of high school mathematics, the literature concerning the mathematical abilities involved in high school physics seems to indicate a need for a clearer and more specific analysis of various terms commonly discussed under the heading of mathematical abilities.

A survey of recent contributions in the field of high school mathematics indicates that certain techniques have been developed in mathematics which can be applied to the solution of this problem.

II. The Problem

Statement of the Problem

Upon the basis of the foregoing summaries of investigations in high school mathematics, it is apparent that there exists a group of abilities having to do with the recognition and use of mathematical concepts, and for the purpose of this study, it is assumed that these

[21] Hinkle, H. C., "Algebra in the Junior High School," *School Science and Mathematics,* Vol. XXV, pp. 271-286.

[22] Christofferson, H. C., "The Teaching of Positive and Negative Numbers," *School Science and Mathematics,* Vol. XXV, pp. 507-514.

abilities have an influence upon performance in physics comparable to that of the group of computational abilities which have been studied by Kilzer, Congdon, and others. It is also assumed that the former group of abilities can be measured objectively.

The major problems of this study, then, may be stated as follows: *To what extent may we expect high school physics students to be able to recognize the mathematical concepts involved in reading and understanding the text and other expositive material in high school physics?*

Two closely related but subsidiary problems are also studied. The first of these is the problem of finding what relationships exist between the ability to recognize the mathematical concepts involved in physics materials and the abilities involved in the solutions of problems of the types found in physics. The second subsidiary problem is concerned with finding the relative importance of two types of abilities, (a) the ability to recognize mathematical concepts, and (b) certain computational abilities, in relation to the success of pupils in high school physics as measured by teachers' marks.

In order to accomplish these purposes it was necessary to find out what mathematical concepts are found in representative physics texts.

Limitations

This study does not attempt to isolate or to measure all of the mathematical abilities that conceivably may be involved in high school physics. In setting up techniques for the measurement of the student's ability to recognize mathematical concepts in physics situations, it may point the way for further analyses.

This investigation does not attempt to measure the ability to use mathematical concepts in the situations occurring in physics except as use is implied in the measurement of certain computational abilities which are considered in this study.

This study does not attempt to deal with all of the mathematical concepts which may be involved to a greater or lesser extent in high school physics. From the nature of the problem it is necessary to limit the list to certain ones of frequent occurrence and to a number suitable for test purposes.

This study is not primarily concerned with current controversies as to the content of the physics course, but attempts to discover facts of value regardless of viewpoints concerning curricular needs in the subject.

No attempt at the validation of mathematical concepts is made since this has already been done by others. For the purposes of this study, Schorling's list of mathematical concepts, which is described in the preceding chapter, is accepted and used since it seems to be the most carefully made list now available.

Procedures Followed in This Study

The testing technique and the statistical procedures involved in the interpretation of test data were the chief methods used in attempting to realize the purposes of this study. As a preliminary step in approaching the major problem, an analysis of three textbooks in high school physics was made. For the purpose of measuring the student's ability to recognize mathematical concepts occurring in physics situations, it was necessary to construct a special research test since no test of this kind was in existence. The other abilities involved in this study were measured by four standardized tests which seemed to be adequate for this purpose.

III. Problems Involved in the Measurement of Mathematical Abilities in High School Physics

Introduction

The chief problems involved in the measurement of mathematical abilities in high school physics as they pertain to this study are as follows:

1. *The definition of the specific abilities to be measured.*
2. *The measurement of the given abilities in physics situations rather than in strictly mathematical situations.*
3. *The construction of a research test to measure the student's ability to react to mathematical concepts in the physics context.*
4. *The preliminary administration of the research test.*
5. *The selection of other research instruments and procedures to be employed.*
6. *The selection of a representative group of subjects for test and study purposes.*
7. *The administration and scoring of the tests and the collection of the necessary data.*

The first five of these problems are discussed in this chapter. The last two are treated in separate chapters since they seem to constitute rather definite units of this study.

The Definition of the Specific Mathematical Abilities to be Measured

The first problem involved in the measurement of mathematical abilities in high school physics is the clear definition of the specific abilities to be measured. At the beginning of this study it was pointed out that many of the preceding studies in this field have defined mathematical abilities in quite general terms rather than in terms of the more specific responses which may be components of these larger abilities. This study attempts a more specific analysis and definition.

It is conceivable that the first specific response that may be involved as a fundamental component of what has been called *reasoning ability* or *problem-solving ability* is the mere recognition of the fundamental idea or ideas included in a given situation. Assuming, for example, that a student has had the necessary experiences for an understanding of proportion, it is probable that when he is confronted by a new situation involving proportion he will be able to proceed much more efficiently if he is quickly able to recognize the element of proportion and to react further to it as direct or as inverse proportion. This reaction may then be the cue or the stimulus to the next reaction in the series necessary to a complete response to the new situation confronting the student.

The first mathematical ability measured in this study is the ability to recognize certain mathematical concepts occurring in high school physics. This presumably involves the type of reaction described in the preceding paragraph, and is in accordance with the rather commonly accepted belief that the first reaction to a new situation is made in terms of elements which it has in common with older and better known situations.

The other group of abilities measured in this study may be called computational abilities. They are concerned with the actual manipulations of mathematical symbols in addition and subtraction of positive and negative numbers, clearing equations of fractions, substitution in formulas, and in other operations of a similar kind. This latter group of abilities is described more completely in the latter part of this chapter.

The Measurement of the Given Abilities in Physics Situations

One of the chief criticisms that can be made regarding many previous studies of mathematical abilities in high school physics is that

they have measured these abilities in mathematical situations rather than in the physics situations in which they occur. Our present knowledge of the facts of transfer of training would lead us to question any conclusions based upon this type of measurement. For this reason the abilities under consideration in this study were measured as far as possible in the actual physics context in which they were found to occur. This fact is brought out in the descriptions of two of the tests used in this study.

The Construction of the Research Test

The test for mathematical concepts in high school physics was constructed for the purpose of measuring the student's ability to react to certain common mathematical relationships and meanings which are involved in the reading and understanding of ordinary physics material from a text or a reference. These mathematical elements are not always expressed in the usual symbols and formulas, and they frequently occur apart from the actual solution of problems.

The first step in the construction of this test was the clear definition of the term *mathematical concept.* This term is briefly described in the preceding pages. For the purposes of this study, a reaction to elements which are present under varying circumstances in many mathematical situations will be called a mathematical concept. Mathematical concepts are interpreted as involving something more than an understanding of the vocabulary of mathematics. For example, the concept of inverse proportion means something more than the ability to recite correctly a definition of the two words or to use them correctly in any given single situation. This concept is used in reacting to varying situations which include the common elements and meanings which are called by this name.

It was then necessary to determine the approximate frequency of occurrence in the physics context of each of the mathematical concepts in Schorling's list which is described in the preceding chapter. For this purpose a preliminary analysis was made of the three textbooks in high school physics which were reported by C. O. Williams as being the most frequently used texts in Missouri high schools in 1927-1928.[1] This analysis included no solutions of problems except

[1] According to an unpublished survey of physics teaching in Missouri conducted by Mr. C. O. Williams, teacher of physics in the Central High School of Kansas City, Missouri, in 1927-1928 these texts were: Millikan, Gale, and Pyle,

the ones which were worked out in the body of the texts to illustrate certain laws or principles of physics. It eliminated such concepts as *net price, simple interest, profit and loss,* and others which do not occur at all in physics. Other concepts in this list were found to occur with varying degrees of frequency. For example, the concept of *direct proportion* occurs many times in the texts under consideration in such statements as:

> If carbon dioxide under a pressure of ten atmospheres is brought into contact with water, ten times as much of the gas is absorbed as if it had been under a pressure of one atmosphere.
>
> Boyle's Law may be explained as follows: Doubling, tripling, or quadrupling the pressure must double, triple, or quadruple the density if the mass remains unchanged.

The concept of *inverse proportion* was found throughout all texts in such statements as:

> The forces of attraction in electrically charged bodies are found, like those of gravitation and magnetism, to decrease as the square of the distance increases.
>
> Experiment shows that in every case of absorption of a gas by a liquid or a solid the quantity of gas absorbed decreases with an increase in temperature.

The concept of *a constant* was found in such statements as:

> The melting point of ice is a perfectly fixed, definite temperature, above which the ice can never be raised so long as it remains ice, no matter how fast heat is applied to it.
>
> The unit of current, the ampere, is the current which will deposit 0.001118 gram of silver in one second.

These illustrations serve to show the form in which mathematical concepts occur in physics material. The next step in the construction of the test was the elimination of certain of the less frequently occurring concepts in order to obtain a list small enough for test purposes.

Criteria for the Selection of Concepts from the Basic List

It soon became obvious that frequency of occurrence of mathematical concepts in physics texts could not be used as the sole basis for deciding what concepts were to be used. For example, the concepts of *length, width, surface,* and *angle* were among the ones of very

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Elements of Physics (Including the earlier *Practical Physics*); Black and Davis, *Practical Physics;* and Dull, *Essentials of Modern Physics.*

frequent occurrence, but were in a form in the physics context judged to be too obvious for testing purposes. For this reason certain criteria for the selection of the concepts to be used were adopted. These criteria may be stated as follows:

1. A given concept, in order to be included in the test, must be common to all three of the texts analyzed.

2. That concept is most acceptable for test purposes which occurs in the greatest number of chapters or sections of the texts analyzed.

3. The concept must not be too obvious in the context in which it occurs.

These criteria were justified upon the basis of practicability as aids in test construction. The final result of the application of the foregoing criteria to Schorling's list was the selection of thirty mathematical concepts to be used in the test. In addition to these, it was found advisable to include the concepts of *constant, limit,* and *identity* which occurred many times in the texts analyzed. These three concepts are listed by one or more of the authorities previously quoted. In the order of their appearance in the test, the concepts thus selected are:

Negative number
Formula
Algebraic factor
Inequality
Direct proportion
Ratio
Constant
Indirect measurement
Approximation in measurement
Perpendicular
Parallel
Minimum
Square root
Coefficient
Equation
Solution of an equation
A power
Exponent
Limit
Identity
Area
Volume
Dimensions
Maximum
Positive number
Graphic representation
Direction
Inverse proportion
Variation
Algebraic product
Algebraic numbers
Equality
Dependence

This list does not pretend to be exhaustive since it was necessary to limit it to a number of items suitable for test purposes. It is believed, however, that it is fairly representative of the mathematical concepts which occur in text and expositive materials in high school physics.

Criteria for the Selection of Test Items

After the completion of the basic list of mathematical concepts to be included in the test, the next step was the selection of representative statements of facts, laws, or principles from physics which include these concepts in context. The following criteria were found to be necessary for this step:

1. A statement or test item that involves a given concept must not contain the word that names the concept. For example, the concept of *direct proportion* occurs in the statement, "The periods of pendulums are directly proportional to the square roots of their lengths." Since this statement contains a rather obvious cue in the words *directly proportional,* it is not as good for test purposes as the statement, "If carbon dioxide under a pressure of ten atmospheres is brought into contact with water, ten times as much of the gas is absorbed as if it had been under a pressure of one atmosphere." The latter statement contains the concept of *direct proportion* without labeling it.

2. A statement selected as a test item must contain a given concept in a valid mathematical form, and must not contain words that are used in a different sense in mathematics. Many statements of fact regarding positive and negative charges of electricity could not be used as illustrations of positive or negative numbers. Likewise statements regarding coefficients of expansion would be confusing because of the different meaning of the term in mathematics.

3. The statement used to illustrate a given concept must be a straightforward statement of fact and not purely hypothetical. Statements involving "If this is the density all the way up, to what height would the atmosphere extend?" are not as desirable for test purposes as more straightforward statements of fact.

4. As far as possible, statements clearly involving more than one mathematical concept should be avoided. Many statements, for example, involve both direct and inverse proportion in the same sentence, and are not desirable as illustrations of either of these concepts since the prepotent element might be *inverse proportion* for one student and *direct proportion* for another.

5. No statement involving too complex a sentence structure or unusual vocabulary difficulties should be used, for the test might then become a test of reading ability.

By means of the foregoing criteria, a large number of statements

of laws, principles, and facts were selected from the three physics texts and labeled as to the mathematical concepts involved. From this list, two statements of similar length were selected for each concept in the basic list of thirty-three which had been previously prepared. In order to obtain a greater uniformity of statement and to avoid unnecessary repetitions of the same fact, all of these statements were quoted from the same text, Millikan, Gale, and Pyle, *Elements of Physics*.[2]

This analysis furnished two lists of thirty-three statements each, and made possible the construction of a test containing two items illustrating each concept. In effect, the test thus constructed contained two forms, each containing the thirty-three concepts of the basic list. For the sake of convenience in tabulation the concepts were arranged in the same order in the two halves of the test. For example, items one and thirty-four, and items two and thirty-five were paired as to the mathematical concept involved. This made it possible for the pupil to make two reactions to each concept and furnished a basis for a study of the nature of difficulty of the mathematical concept as it appears in the physics context. It was also believed that more reliable conclusions would be made possible by this method than would be the case if the student were allowed but one reaction to each concept.

After some experimentation with various types of test forms the multiple-choice form was chosen. Each of the statements selected for the test is followed by the names of five concepts from which the pupil selects the one that is involved in the accompanying statement.[3]

The Preliminary Administration of the Test

The test was then mimeographed and administered in April, 1930, to a total of 162 physics students in six Missouri high schools.[4]

Although the results from the preliminary administration of the test revealed the need for certain revisions, it was found to have a fairly satisfactory reliability as indicated by the application of the

[2] Millikan, R. A., Gale, H. S., and Pyle, W. R., *Elements of Physics*, Ginn and Company, 1927.

[3] See the complete test form.

[4] The schools cooperating in the preliminary testing were Boonville, Centralia, Mexico, Moberly, Columbia, and the University High School.

self-correlation technique and the Spearman-Brown formula[5] which gave a reliability coefficient of .88 (P.E. .012).

The total number of errors on each test item was then tabulated for the 162 cases and comparisons as to difficulty were made for each of the thirty-three pairs of concepts. These comparisons showed that a few items were too difficult or too easy because of the length or complexity of the statement, or because of an obvious or ambiguous list of responses. When this analysis showed that a given test item might be improved, it was either changed or discarded in the final revision of the test. No test item was changed, however, merely because of its ease or difficulty.

The research test developed as a result of this preliminary study was called the *Carter Test for Mathematical Concepts in High School Physics* and will hereafter be called by that name. This test was designed to measure the student's ability to recognize the mathematical concepts occurring most frequently in the reading of expositive materials of high school physics. It does not involve computations or the manipulation of mathematical symbols. It differs from a vocabulary test in that the student is not required to make responses to word definitions, but responds to a complete sentence or paragraph by selecting from a list of five possible answers the one correct response which is the name of the concept involved in the test item. This point will be made clearer by a reference to the complete test form.[5a]

Objectivity of the Test

As indicated in the foregoing paragraph, the research test is very objective in the type of response required of the pupil. It is so constructed that but one of the five possible responses listed under each item can be right. The student is to make only one response to each item and he is to make that response by deciding which one of the five choices is correct and then placing the number of that choice in the parentheses at the right of the item. The test is scored by means of a key and one point is allowed for each correct reaction.

[5] Garrett, H. E., *Statistics in Psychology and Education*, Longmans, Green, 1926. pp. 268-271.

[5a] Carter, W. R., *Carter Test for Mathematical Concepts in High School Physics*, University of Missouri.

The student's score is the number of correct responses. The correctness of the keyed responses to each item was obtained by a careful analysis of the results from the preliminary administration of the test and consultations with a member of the mathematics department of the University of Missouri and with a teacher of mathematics with considerable experience both in the teaching of mathematics and in test construction. It is believed that a high degree of objectivity has been obtained by these techniques.

Validity of the Test

Care was used in the construction of the test to see that it met certain requirements as to validity. As has been pointed out in the preceding pages of this chapter a basic list of concepts was used which has been agreed upon by a number of authorities. The test items involving these concepts were quoted directly from a widely used text in high school physics. It was believed that this would insure a reasonable degree of validity. Further evidence on this point is presented in Part VI in connection with the discussion of the results from the final administration of the test.

Reliability of the Test

The application of the self-correlation technique and the Spearman-Brown formula to the results of this test in its final form as administered to 526 physics students gave a reliability coefficient of $.88 \pm .007$. This indicates that the test has a fair degree of reliability in its present form.[6]

Other Research Instruments

In order to obtain a further measure of students' knowledge of mathematical concepts and to check the results on the research test, the Butler Test for Mathematical Concepts in Junior High School Mathematics[7] was used. This test measures the student's ability to recognize mathematical concepts in mathematical situations rather than in physics situations and is based on the same list of concepts as the Carter test, twenty-three of them being common to both tests.

[6] Garrett, H. E., *Op. cit.*, pp. 268-271.

[7] Butler, C. H., "The Butler Test for Mathematical Concepts, Junior High School," 1930. Published privately by the author at Columbia, Missouri. See also THE MATHEMATICS TEACHER, March, 1932, pp. 117-172.

The second type of abilities measured in this study has been called computational abilities, or those abilities having to do with the actual manipulation of mathematical symbols and the use of certain items of mathematical information in the solution of problems of the kind usually found in high school algebra and occurring in the solution of problems in physics. For the measurement of this group of abilities there exists a well-standardized test, the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics, Part I.[8] This test purports to measure the student's ability to handle the algebraic operations or manipulations that occur most frequently in the solutions of physics problems. It includes such operations as the addition of negative numbers, clearing equations of fractions, and factoring binomials. According to the author, it was constructed after a careful solution and analysis of all of the problems in the five most frequently used physics texts and manuals in 345 Iowa high schools in 1927-1928.[9]

Other abilities that might have a decided influence upon performance in physics are intelligence and reading ability. In order to give proper consideration to these abilities and to make certain analyses and comparisons, the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B,[10] and the Nelson-Denny Reading Test for Colleges and Senior High Schools[11] were used.

[8] Kilzer, L. R., and Kirby, T. J., "An Inventory Test for the Mathematics Needed in High School Physics," Public School Publishing Company, Bloomington, Ill. 1929.

[9] Kilzer, L. R., and Kirby, T. J. *op. cit.*, Manual, p. 2.

[10] Otis, Arthur S., "Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B." World Book Company, 1928.

[11] Nelson, M. J., and Denny, E. C., "The Nelson-Denny Reading Test for Colleges and Senior High Schools, Form A." The Houghton Mifflin Company, 1930.

Properties of Squares and Cubes of Arithmetical Numbers

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Somewhat over a year ago a joke about an eccentric professor made the rounds of nearly every periodical and newspaper of the country. The joke told of a mathematics professor testifying in a police court as a witness of an auto accident. The professor could not remember the license number of the car, but he did remember that he had noted the peculiar coincidence that if the digits of the license number were reversed, and if this result were multiplied by 50, then the sum of the digits would be exactly equal to the cube root of this product.

Wishing to be of assistance to the perplexed police, the writer thought this circumstantial evidence should be sufficient to lead to the apprehension of the reckless driver. It should be an easy matter to find the license number from the given data.

In the first place the characteristics of the final product had to be examined. This product had to end in a zero on account of the multiplication by 50. Then, since this product was a perfect cube, it would have to end in three zeros. But license numbers usually do not begin with a zero. How then could the last digit of the reversed number be a zero? Here was a hitch. How could you get a product ending in three zeros upon multiplying by 50 unless the reversed number ended in at least one zero? In the next place it was assumed that a license number might begin with one or even two zeros. But even then no number could be found which satisfies the given conditions, in fact it soon became apparent that no such number exists.

The writer surmised that perhaps the court reporter had slipped up, and that the professor had said nothing about the multiplication by 50, but had reversed the order of the digits, and had then found that the cube root of the reversed number was equal to the sum of the digits. In this case there are really five numbers which fulfill the given conditions. The license number then might have been 215, 3194, 2385, 67571 or 38691. Reversing the digits respectively gives: 512 whose

cube root is 8, 4913 whose cube root is 17, 5832 whose cube root is 18, 17576 whose cube root is 26, and 19683 whose cube root is 27.

The search for numbers where the sum of the digits is equal to the cube root is naturally restricted by the number of digits in a cube relative to the number of digits in its cube root. This relationship disqualifies all numbers whose cube root is greater than 54, and this last number could qualify only if the cube root of 999,999 were actually 54. The examination of eligible cubes then is quite restricted.

It might be mentioned further that only one number exists whose square root is equal to the sum of its digits, namely 81, while there are five numbers whose fourth roots are equal to the sum of the digits, namely, 2401, 234256, 390625, 614656, and 1679616. The fourth roots of these numbers are 7, 22, 25, 28, and 36, respectively. The comparative number of digits bars squares whose square roots are above 18, and double squares whose fourth roots are above 72 from qualifying for such unique distinction.

Since cubes and cube roots have been haled into court it might not be without purpose to submit them and their younger brothers, namely squares and square roots, to further and somewhat detailed examination. The properties of cubes and squares, though generally known,[1] may nevertheless be of interest, and could serve profitably in supplying material for programs in mathematics clubs of high schools.

In discussing squares and cubes the field may be divided for convenience into six sections:

I. Comparative Properties of Squares and Cubes.
II. Relation of Numbers to Squares and Cubes.
III. Relation of Sequences to Squares and Cubes.
IV. Relations between Squares.
V. Means of a Series of Squares.
VI. Additional Properties of Cubes.

These divisions are merely for convenience. They are not mutually exclusive nor necessarily logical. However, they serve as a more or less natural grouping for a piece-meal analysis of typical properties of squares and cubes. In the following discussion number is limited to those of arithmetic, namely to positive integers.

[1] See Davis and Peck, *Mathematical Dictionary and Cyclopedia*, A. S. Barnes & Co., New York, 1855. (Cf. "Squares and Cubes of Numbers.") See also exercises in Carmichael, *Theory of Numbers*.

I. Comparative Properties of Squares and Cubes

1. *Termination of squares and cubes.*—From a cursory inspection of a table of squares and cubes it is apparent that a number cannot be a square unless it ends in 0, 1, 4, 5, 6, or 9, while a cube may end in any number whatever. However there is a natural sequence in the ending of successive squares and cubes, and for that matter for any successive powered numbers. These sequences have a periodical recurrence and are as follows:

Squares	1	4	9	6	5	6	9	4	1	0
Cubes	1	8	7	4	5	6	3	2	9	0
Fourth Power ...	1	6	1	6	5	6	1	6	1	0
Fifth Power	1	2	3	4	5	6	7	8	9	0

The successive powers, higher than the fifth, repeat the above sequences in regular order, namely, the sequence for the sixth power is the same as that of squares, for the seventh power the same as that of cubes, etc.

2. *Identifying squares.*—A simple test, though negative, may be applied to rule out those numbers which are not squares. A square can not end in two even digits nor in two odd digits. An exception to this rule occurs when a square ends in 4. In the event a square ends in 4 the last two digits must be even numbers. It should be noted here that zero is considered neutral, and should not be classed definitely as an odd or even number.

3. *Form of squares and cubes.*—Every square number is of the form $4n$ or $4n+1$. Hence if any square number is divided by 4 the remainder must either be 0 or 1. Similarly every cube number is of the form $4n$, or $4n\pm1$. Consequently the remainder of a cube upon division by 4 must be 0, 1, or —1. In the above forms for squares and cubes 4 is called the *modulus* of the number. A variety of forms can be found resulting in more or less restricted forms by using other numbers as moduli. A particularly restrictive form of cubes is found when 9 is used as the modulus, in which case the form is $9n$ or $9n\pm1$.

4. *Ultimate differences of powered numbers.*—If we take the differences between the successive numbers of a regular sequence of squares, and in turn again take the differences of the successive numbers of the resulting sequence then the second differences will equal 2. This property can readily be verified by trial. In the following illustration the numbers of the second and third row are the differences of the two numbers immediately above them.

Squares	4		9		16		25		36		49		64
1st diff.		5		7		9		11		13		15	
2nd diff.			2		2		2		2		2		

Similarly the third differences of successive cubes are equal to 6, viz:

Cubes	8		27		64		125		216		343		512
1st diff.		19		37		61		91		127		169	
2nd diff.			18		24		30		36		42		
3d diff.				6		6		6		6			

(It may be noted here that this remarkable property of ultimate differences is perfectly general for the sequence of numbers of any power. Perfectly general the nth difference of a series of the nth power numbers is $1.2.3.4 \ldots n$ or $n!$)

II. Relation of Numbers to Squares and Cubes

1. *Every prime number of the form* $4n+1$ *and every power thereof can be expressed as the sum of two squares.*[2]—Take, for example, the prime numbers 5, 13, 17, 29, 37, and 41. Checking the property up to the third power gives results as follows:

$$\begin{array}{lll}
5 = 1^2 + 2^2 & 5^2 = 3^2 + 4^2 & 5^3 = 5^2 + 10^2 \\
13 = 2^2 + 3^2 & 13^2 = 5^2 + 12^2 & 13^3 = 9^2 + 46^2 \\
17 = 1^2 + 4^2 & 17^2 = 8^2 + 15^2 & 17^3 = 47^2 + 52^2 \\
29 = 2^2 + 5^2 & 29^2 = 20^2 + 21^2 & 29^3 = 65^2 + 142^2 \\
37 = 1^2 + 6^2 & 37^2 = 12^2 + 35^2 & 37^3 = 107^2 + 198^2 \\
41 = 4^2 + 5^2 & 41^2 = 9^2 + 40^2 & 41^3 = 115^2 + 236^2
\end{array}$$

Whereas the two relations, in which the primes[3] as well as their squares[4] are the sum of two squares, are unique, that in which the sum of two squares is a cube admits of other solutions. For example: $13^3 = 13^2 \times 13$. But from the table above $13 = 2^2 + 3^2$, whence $13^3 = 13^2(2^2+3^2) = 26^2+39^2$.

2. *Every odd number can be expressed as the difference of two*

[2] Fermat's ***Diophantus***, Toulouse, 1670, bk. III., p. 127; Brassinne's ***Précis***, Paris, 1853, p. 65.—Cf. Ball, ***Mathematical Recreations and Essays***, 1922, p. 36. See also T. L. Heath, ***Diophantus of Alexandria***, ed. 2, 1910.

[3] Theorem by Fermat: "A prime in the form $4n+1$ can be expressed once, and only once, as the sum of two squares." Cajori, ***A History of Mathematics***, 1919, p. 168. See also T. L. Heath, op. cit.

[4] Theorem by Fermat: "A prime in the form $4n+1$ is only once the hypothenuse of a right triangle; its square twice; its cube three times, etc. Examples: $5^2=3^2+4^2$; $25^2=15^2+20^2=7^2+24^2$; $125^2=75^2+100^2=35^2+120^2=44^2+117^2$." Cajori, op. cit., p. 168. See also T. L. Heath, op. cit.

squares.—Every odd number is of the form $2n+1$. Evidently $2n+1 = n^2+2n+1-n^2 = (n+1)^2-n^2$. Examples: $3 = 2^2-1^2$, $9 = 5^2-4^2$, $11 = 6^2-5^2$, etc.

$a+b=2n$
$a-b=2$
$a=n+1$
$b=n-1$

3. *Every even number of the form* 4n *can be expressed as the difference of two squares.*[5]—It is readily apparent that $4n = n^2+4n+1-n^2-1 = (n^2+2n+1)-(n^2-2n+1) = (n+1)^2-(n-1)^2$. Examples: $12 = 4^2-2^2$, $28 = 8^2-6^2$, etc.

III. Relation of Sequences to Squares and Cubes[6]

1. *The product of 4 consecutive numbers plus 1 is a perfect square.* Let n, $n+1$, $n+2$, $n+3$, be the four consecutive numbers. The product of these four numbers increased by unity is $n^4+6n^3+11n^2+6n+1$. Since this expression factors into $(n^2+3n+1)^2$ the property is proven.

2. *The sum of* n *consecutive odd numbers from* 1 *to* 2n—1 *is equal to* n^2.[7]—In the language of algebra this means:

$$1+3+5+7+9+ \text{— — — — — — — —} + 2n-1 = n^2$$

By actual trial we can establish this relation for a series of as many terms as we please. The actual proof is by mathematical induction and will not be given here.

3. *The sum of an arithemetical sequence of* n *terms, with a common difference 2, in which the first term is* n^2—n+1 *is equal to* n^3.[8]—This property is readily established by remembering that there are n terms in the series. The sum then becomes:

$$n\times n^2-n\times n + (1+3+5+ \text{— — — —} +2n-1)$$

Noting the equivalence of the series in parenthesis from the foregoing proposition, the sum can be written: $n^3-n^2+n^2$. Since this is equal to n^3 the proposition is established. Examples: $13+15+17+19 = 4^3$, $21+23+25+27+29 = 5^3$. starting from 1

4. *The sum of any number of consecutive cubes* starting from 1 *is the square of the sum of the cube roots.*[9]—This proposition, like proposition 2 above,

[5] This relation is not based on the formula by Plato: $(2m)^2=(m^2+1)^2-(m^2-1)^2$. —Cf. Heath, *History of Greek Mathematics*, 1921, v. I., p. 81.

[6] Properties 2, 3 and 4 of this section are usually included as illustrative exercises for mathematical induction in texts on college algebra.

[7] This relation was known to Pythagoras. Cf. Heath, *History of Greek Mathematics*, v. I., p. 77.

[8] Theorem by Nicomachus. Cf. Heath, *History of Greek Mathematics*, v. I., p. 108.

[9] First stated and proved by al-Karkhî of Bagdad in the beginning of the eleventh century. Cf. Cajori, op. cit., p. 106.

depends for proof on mathematical induction. By actual trial however, the relation can be verified for as many terms as desired, viz: $1^3+2^3+3^3+4^3+5^3=(1+2+3+4+5)^2$.

IV. Relations between Squares

1. *The sum of two odd squares cannot be a square.*—Since every square number may be expressed in the form $4n$ or $4n+1$, it follows that all odd squares are of the form $4n+1$. Let $4p+1$ and $4q+1$ be two given odd squares. The sum of these two odd squares is $4(p+q)+2$. This sum can not be a square since it is neither of the form $4n$ nor $4n+1$.

2. *An odd square taken from an even square cannot leave a difference which is a square.*—Let $4p$ be an even square and $4q+1$ an odd square. Taking the odd square from the even square leaves $4(p-q)-1$. This result can not be a square since it is neither in the form $4n$ nor $4n+1$.

3. *If three numbers* x, y, *and* z, *are so related that* $x^2+y^2=z^2$, *then* $x=2pq$, $y=p^2-q^2$, *and* $z=p^2+q^2$.[10]—This proposition can be verified by substituting the values given for x, y, and z, in the equation $x^2+y^2=z^2$. Since the values of x, y, and z depend directly on values of p and q, the above relations suggest a simple method for finding examples of right triangles for which the length of the sides can be represented by integral numbers.[11]

To illustrate:

p	q	x	y	z	p	q	x	y	z
2	1	4	3	5	4	3	24	7	25
3	2	12	5	13	5	1	10	24	26
3	1	6	8	10	5	2	20	21	29
4	1	8	15	17	5	3	30	16	34
4	2	16	12	20	5	4	40	9	41

4. ***If the sum of the squares of two numbers is the square of a third number, then one of the numbers is divisible by 3, one (possibly the same one) is divisible by 4, and one is divisible by 5.***—The validity of

[10] J. W. A. Young, ***Monographs on Topics of Modern Mathematics***, 1915, p. 318. See also Carmichael, ***Diophantine Analysis***, 1915, p. 10.

[11] Three positive integers x, y and z, satisfying the relation $x^2+y^2=z^2$ are said to form a Pythagorean triangle. Pythagoras, Plato, Diophantus, and others gave formulas for finding such sets of numbers. cf. Heath, ***History of Greek Mathematics***, v. I., pp. 80, 81.

this proposition can be tested for the sets of x, y, and z, of the foregoing table. For proof of this proposition the reader may refer to *Monographs on Modern Mathematics* edited by J. W. A. Young.[12]

5. *The difference of two consecutive squares is equal to twice the square root of the lesser increased by 1.*—In the language of algebra this would read: $(n+1)^2—n^2=2n+1$. This result is known as the first difference of two consecutive squares.

V. Means of a Series of Squares

1. *The mean proportional of any two squares in a series of squares of natural numbers, is equal to the lesser square plus the product of its square root by the difference of the square roots of the two numbers.* If the two given squares are m^2 and $(m+n)^2$, then the mean proportional of these two numbers $m(m+n)$ or m^2+mn. Since n is the difference of the square roots of the two numbers the proposition stands.

2. *The arithmetical mean of any two squares exceeds their geometrical mean by half the square of the difference of their square roots.*— The arithmetical mean of the two squares m^2 and $(m+n)^2$ is $\dfrac{m^2+(m+n)^2}{2}$. Algebra shows that this mean exceeds m^2+mn, the geometrical mean of these two squares, by $\dfrac{n^2}{2}$ This establishes the proposition.

3. *Of three equidistant squares in a series of consecutive squares the geometrical mean of the extremes is less than the middle square by the square of the difference of the square roots of the given squares.* —Let $(m—n)^2$, m^2, and $(m+n)^2$ be three equidistant squares. The geometrical mean of the extremes $(m—n)^2$ and $(m+n)^2$ is $(m—n)(m+n)$ or $m^2—n^2$. But this is the middle square m^2 less the square of n, the difference of the square roots of two of the three equidistant squares.

VI. Additional Properties of Cubes

1. *If the cube of a number is divided by 6 the remainder will be the same as that of the number itself divided by 6.*—The above property must be valid if the difference of the cube and the number is exactly divisible by 6. If n is the given number and n^3 its cube, then the difference is $n^3—n$. This expression factors into $(n—1)n(n+1)$. It must

[12] J. W. A. Young, op. cit., p. 319.

be noted that these factors are three consecutive numbers. Of any group of three consecutive numbers, at least one of the numbers is divisible by 2 and one by 3. From this it follows that the product of three consecutive numbers is divisible by 6, and the proposition is established.

2. *Neither the sum nor the difference of two cubes can be a cube.*[13] —Since there are an unlimited number of cases where the sum of two squares is also a square it might be expected that there should likewise be cases where the sum, and consequently also the difference, of two cubes is a cube. But no such set of numbers has ever been found, nor ever can be found as was shown by Euler over 150 years ago.[14]

With the above observations the case against squares and cubes will be dismissed. The listing of properties is by no means complete.[15] Interesting extensions could be made by finding sets of more than two numbers where the sum of their squares is also a square, viz: $1^2+2^2+2^2=3^2$, or by finding sets where the sum of two squares is equal to the difference of two other squares, as $2^2+6^2=7^2-3^2$. Search for such and similar relations should furnish helpful stimulus for mathematics classes, and attempts to prove such discovered relations, where susceptible to simple proofs by means of algebra, gives the student a new field for a new tool. At the same time such investigations might give the student some appreciation of the power algebra furnishes him to establish proof where, without algebra, proof would seem quite hopeless.

[13] This is a special case of Fermat's Last Theorem, viz: "If n is an integer greater than 2 there do not exist integers x, y, z, all different from zero, such that $x^n+y^n=z^n$." Fermat noted this theorem on the margin of a book and added that he had discovered a marvelous proof thereof, but that the margin was not large enough to contain the proof. In 1908 a bequest of 100,000 marks was made as a prize for a complete proof of this theorem before 2007. Proofs for values $n<7000$ have been given. Cf. Smith, *Source Book in Mathematics*, p. 213; Cajori, *A History of Mathematics*, pp. 168, 442; Ball, *Mathematical Recreations and Essays*, pp. 40-43. For a recent contribution see *Science News Letter*, Sept. 19, 1931.

[14] J. W. A. Young, *Monographs on Topics of Modern Mathematics*, p. 319; Cajori, *A History of Mathematics*, p. 168.

[15] Interesting material of this nature may be found in Dickson, *History of the Theory of Numbers*. See also Heath, *Diophantus of Alexandria*, ed. 2.

A Course in Mathematics for Pupils Not Going to College

By Virgil S. Mallory, *Associate Professor of Mathematics State Teachers College at Montclair, New Jersey*

Eight years ago I became interested in organizing a course in mathematics for two classes of pupils: those who elect no mathematics in high school and those who, having elected the regular work, soon find themselves beyond their depth. I am frank in confessing that the original motive was more a desire to improve the teaching of the better students than to aid those in the poorer group. The reasons that led me to do this were probably the same reasons that have led others to establish similar courses in many high schools throughout the United States. Some of these reasons follow:

1. A desire to provide an enriched course in mathematics for students of superior ability.
2. An attempt to reduce the percentage of failures in college preparatory courses in mathematics.
3. An attempt to make certain that the poorer group were taught material within their understanding and interests.
4. An attempt to provide for those pupils who remain in high school only one year a unified course which will present those elements in mathematics which are of most value, which are most easily understood, and which are of the greatest interest.
5. A belief that large numbers of pupils go through high school with no mathematics who will benefit if a course is provided which offers a reasonable chance of success.
6. A belief that a properly selected and graded course in mathematics offers elements of profit to every student who attends high school.

It must be seen that my conception of this course involves two general principles. First, enrollment of all high school pupils in a course in mathematics fitted to their abilities and interests, and second, the enrichment of the course in mathematics for the better students so that time they now spend in excessive drill may be given to the consideration of mathematical topics not generally included in high school courses.

Amid the changing conditions in secondary education the necessity for such a new course has been widely recognized. Fifty years ago the high school had as its definite task the preparation of all students for college work. It is undoubtedly true that the mental caliber of the average pupil then was higher than that of the average pupil in high school at the present time. All educational authorities have recognized that this change in the secondary school population should necessarily change the aims of secondary school instruction. Despite this fact secondary schools generally provide a single course in mathematics, that which provides for college preparation, and other pupils are discouraged from taking mathematics. Still there is the demand that the course in mathematics must be made so easy that every student can successfully pursue it.

Thus, since large numbers of high school pupils are incapable of doing the regular work in mathematics an excessive number of failures is prevented by ignoring the bright pupil and attempting to dilute the regular work to fit the mental capacities of the mediocre pupil. Thus we have excessive drill on manipulation in algebra far beyond the needs of the bright pupil and find that even then the results obtained from the poorer group are disappointing. It is doubtful if a year spent in an attempt to acquire manipulative skill in factoring, solving puzzle problems, and working with fractions and radicals is of much value to these pupils. Nor, if they succeed in barely getting by in the first year's work, is it certain that another year spent in memorizing geometric definitions and theorems will be of much value to them. For this type of pupil, having no great capacity for original thinking, must either slow up the brighter pupil to his pace or else go through the course in geometry so rapidly that he has gained little of value at the completion of the course.

This does not mean that there are no topics in algebra and geometry which have value for these pupils. It does mean that the material from algebra and geometry to be included in a course for them must be selected with more appreciation of their capabilities and interests. In this way only can we present real courses in mathematics for those pupils who have the intellectual ability to enjoy them.

Thus, in our attempts to compromise on a single course for all pupils, we are failing to adequately provide for either group. On the one hand, achievement tests tell a sorry story of the lack of

success of the poorer pupils in mastering the simplest elements of the mathematics taught, while, on the other hand, we find the brighter pupil wasting time on excessive drill work which he does not need. Finally we acquiesce in the decision of the general educator that while history and English and general science is good for the large number of pupils in the general course, there is nothing of value to him in a course in mathematics.

For the fact that many high schools present but a single course in mathematics—that which prepares for college—means that many pupils elect no mathematics at all or else pursue the college preparatory course with doubtful success. There is grave doubt whether pupils who have spent a year in drilling on the ability to manipulate algebraic symbols and who fail or obtain low marks have acquired very much of value in their education.

In cities throughout the United States this situation has been met in a number of different ways:

1. Generally, the college preparatory course in mathematics is the only course presented. Pupils without the mental ability requisite for success in this course are discouraged from taking it or receive failure marks or low marks if they insist on taking it. Naturally their presence in the class lowers the standard of work. Thus teachers everywhere are interested in devising methods of determining in advance the probable success of pupils in algebra and geometry.

2. In some cities, either by means of prognostic tests or of marks in seventh and eighth grade arithmetic, or by a trial short course in algebra, the weaker pupil is put in a separate class and is given a diluted course in the regular college preparatory mathematics. The method of diluting the course is either by lengthening the time requisite for the completion of such a course, or else by cutting down on the number of pages in the textbook which are presented.

3. In many places pupils are given courses in arithmetic which are largely a review of the arithmetic of the seventh and eighth grades.

The second and third of these methods recognizes the economic loss of failures in mathematics and clears the way for better work for students of higher intelligence. None of them, however, takes into account the special needs and interests of this group. While a first course in algebra offers many topics of value to these pupils it is doubtful if a mere dilution of the regular course is the best solution of the problem.

The published results of tests show that these pupils either through mental immaturity or lack of ability, have obtained neither a desirable mastery of computation nor a clear understanding of simple mathematical relationships. Still a course made up largely of a review of seventh and eighth grade mathematics is not the best course for them for two reasons. First because a review course will fail to arouse that interest and enthusiasm which is necessary to success, and second because the course ignores many elements vital to the pupil's future needs. For whatever his vocation, in every walk of life, he will continually meet the necessity of interpreting mathematical relationships either in the definite number of arithmetic or in the general number of algebra. In a similar manner a course in industrial or so-called shop mathematics is not satisfactory because it is too highly technical and too little concerned with general principles. The specific requirements of any particular trade can be easily acquired by the pupil who appreciates and understands mathematical relationships. Finally since in whatever the pupil does in future life he must constantly meet mathematical relationships and make decisions influenced by them he must not be allowed to go through high school without a training which holds such immediate values for him. As teachers of secondary school mathematics we should be ashamed to imply that we cannot devise a course in a subject so vital to his future economic and cultural life.

Hence any such course in mathematics for these pupils should be based on three general principles:

1. The material presented should be that which will definitely function in the life of the pupil. Thus, because he may take but one year of mathematics he should study mathematics more extensively than intensively.

2. It must be material of a type which will interest him through convincing him of its practical values. It is impossible to do this if the work is of exactly the same type that he has had before. While he needs further drill in ordinary computation and in simple quantitative relationships this material must be presented in a new setting and in a way that convinces him that his knowledge is increasing. Continued drill on earlier work without new motivation will soon meet the law of diminishing returns.

3. The material must be within the pupil's capacity to understand. Not only must this be borne in mind in the selection of material but also in the method of presentation. Explanation must be much

clearer than with the better pupils. Topics once learned must be frequently reviewed and emphasized in new relations. Finally every possible means of pupil activity must be adopted in order to emphasize the reality of the problem the pupil is working. While the bright pupil reads and solves in the classroom a problem dealing with indirect measurement, to many of the poorer pupils it means nothing in real life at all, and is only solved by following a set of rules meaningless to him.

It is impossible at this time to take up in detail all of the problems which must be solved in devising such a course. The most important concern the objectives to be sought, the content of the course of study, the methods of teaching, and a study of those types of pupil activity which can be used.

Desirable objectives can never be determined by asking the man in the street who knows no mathematics what to teach. The answer is rather to be found by a consideration of what mathematics the pupil could use if he thoroughly understood it.

The determination of material for such a course depends on the objectives sought. However, a list which is not exhaustive might be:

The functional relation
Statistical and algebraic graphs
The general number
The formula
The equation
Intuitive geometry
Indirect measurement
  By scale drawings and similar figures
  By trigonometry
A unit of demonstrative geometry
Geometric constructions
Social arithmetic
The use of tables
The slide rule

Not all of these topics satisfy all three of the criteria laid down for organizing such a course. Some of them are included because they are of value in motivating the work and in providing a means of encouraging pupil activity. Thus the pupil learns how to use the slide rule, not necessarily because it is a tool which he will use after leaving school, but because it is a device which is interesting, which he enjoys using, and which in its use gives him drill and practice in

accurate computation, in the use of formulas, and in the understanding of quantitative relationships. For the same reason he makes and uses a simple transit, and finds the height of tall trees, the width of a river, and the distance away of an inaccessible object. Again it is not thought that these particular problems will be a part of his future life work, but it is hoped that they will arouse his interest in mathematical thinking and that they will convince him of the practical values of mathematical studies. The drawing board and T-square are a part of the equipment of the mathematics room because they are an aid in the development of neatness and accuracy and because they provide another means of pupil activity. Thus his interest in mathematical studies is increased and he is more willing to spend time in perfecting his knowledge of mathematical relationships.

The present experiment that is being conducted in six New Jersey high schools is an attempt to answer some of the many questions which the earlier part of this paper has suggested. Thus,

1. What objectives and outcomes should be sought?
2. What topics in mathematics most readily function in the life of the pupil?
3. What should be the order in which this material should be taught?
4. What types of teaching will insure best retention on the part of the pupil?
5. What types of pupil activity can be utilized in arousing interest and encouraging mathematical curiosity in these pupils?
6. What types of life situations will appeal to the pupil as practical?

These are only a few of the many questions that arise. With the foregoing general principles in mind and in an attempt to answer some of these questions schools in Elizabeth, Englewood, New Brunswick, Dover, Kearny, and Hasbrouck Heights have furnished classes totaling 200 pupils. The schools have provided these pupils with materials and instruments which tend to encourage pupil activity. In return I have supplied in lithoprinted form units of work in a number of topics in algebra, geometry, trigonometry, mathematical drawings, etc. No claim is made that this material is the best that could be provided, for the purpose of the experiment is to determine what material is best.

In determining experimentally the content of the course the fol-

lowing precedure is adopted. Certain units of work have been selected from the field of mathematics whether or not they have previously been taught in the regular four year high school course. After being taught a unit of work the pupil is tested. It is hoped that the results of these tests will show how much each of these elements contributes to the pupil's interest in mathematics, to his acquisition of further skill in computation, and to his ability to see and understand quantitative relationships. In the selection of this material topics which stress practice in algebraic manipulation have been omitted. Thus factoring and work with complicated fractions is not taken.

In the language of the pupil I have attempted to furnish the initial motivation in the Foreword to the text which is put into his hands:

> This course in mathematics that you are going to take is a new one. It is made up of the most interesting topics from the regular four year course in high school mathematics. While the other freshmen in the mathematics classes will study algebra all of this year, you will learn about several topics that they will not learn about until later. Thus you will learn, among other things,
>
> 1. Why algebra is important to you and how it is used.
> 2. Why geometry is important to you and how it is used.
> 3. Why trigonometry is important and how it is used.
> 4. How to make and use an instrument which will enable you to find the height of a tree, the length of a pond, etc. This instrument is used by surveyors and is called a transit.
> 5. How to make and use an instrument which will multiply and divide. It is used by engineers and is called a slide rule.
>
> There are many other topics in the course and I hope that you will like them all.
>
> You know, however, that nothing worthwhile can be obtained without hard work. Hence you will have to listen carefully and study hard in order to understand and enjoy the interesting things that you will learn.

There is no claim at the present time that the ideal material has been selected and is being presented. It is hoped, however, that through this experiment it will become definitely known what material in mathematics is of most value to and is not beyond the comprehension of the pupil.

The work on this experiment this year has been largely to determine the lines of the complete experiment. It is planned next year to continue the same work with many revisions in both tests and materials of instruction and with a greater number of classes.

Thirteenth Annual Meeting of the National Council of Teachers of Mathematics

Washington, D. C., February 19-20, 1932

Directors' Meeting, February 19, 10:15 a.m.
English Room, the Raleigh Hotel

PRESENT: John P. Everett, Martha Hildebrandt, Edwin W. Schreiber, William D. Reeve, Vera Sanford, J. O. Hassler, C. M. Austin, Harry C. Barber, William Betz, Marie Gugle, Mary S. Sabin. Absent: W. S. Schlauch, Herbert E. Slaught, John R. Clark, Elizabeth Dice, C. Louis Thiele.

The following business was transacted. The report of the secretary-treasurer as treasurer was read, discussed, and accepted. The report of "Committee on Sale of Yearbooks and Extension of Council" was submitted by the chairman, Mr. Austin, and accepted as read. The bills of Mr. Austin's committee were approved and the treasurer instructed to pay same. Voted that the committee on "Sale of Yearbooks and Extension of Council" be continued with a maximum appropriation of three hundred dollars. Voted that the treasurer be instructed to pay Mr. Reeve sixty dollars as one-half the expense of advertising THE MATHEMATICS TEACHER and yearbooks. Voted that the "Committee on Official Journal" and "Committee on Sale of Yearbooks and Extension of Council" be empowered to settle the price of past yearbooks. Voted that the treasurer be instructed to send fifty dollars to Professor David Eugene Smith for clerical work in connection with the Report of the International Congress on the Teaching of Mathematics.

Directors' Meeting, February 19, 2:00 p.m.
English Room, the Raleigh Hotel

Present: John P. Everett, Martha Hildebrandt, Edwin W. Schreiber, William D. Reeve, Vera Sanford, John R. Clark, J. O. Hassler, C. M. Austin, Harry C. Barber, William Betz, Marie Gugle, Mary S. Sabin, C. Louis Thiele. Absent: W. S. Schlauch, Herbert E. Slaught, Elizabeth Dice.

Voted that the chair appoint a committee of three to report on the matter of directors' expenses to annual meetings. President Everett appointed Miss Sabin, Mr. Austin, Mr. Reeve, as members of this committee. Voted that the directors' dinner for Friday evening be cancelled. Voted that the treasurer be paid one hundred dollars from the fund appropriated for supplies to take care of the additional expense of bringing the secretary's records up to date. Voted that four copies of the mailing list of THE MATHEMATICS TEACHER for March be sent as follows: one copy to the president's office; one copy to the secretary-treasurer's office; one copy to the chairman of the "Committee on Sale of Yearbooks and Extension of the Council"; the fourth copy to be divided into parts by states and distributed among the Board of Directors according to their geographic location. Voted that the secretary be instructed to purchase complete files of THE MATHEMATICS TEACHER and *School Science and Mathematics;* same to be placed as a permanent possession of the secretary-treasurer's office. Voted that the chair appoint an official delegate for the meeting of the International Congress on the Teaching of Mathematics to be held in Zurich, Switzerland, in September, 1932. President Everett appointed Mr. Reeve as delegate with the proviso that if he could not attend he should secure an alternate. Voted that the present Committee on Ballot continue the present plan, incorporating the suggestions made by Miss Sanford, concerning an appeal for prospective candidates from affiliated organizations. Voted that Mr. Betz act as official spokesman for the Board of Directors at the breakfast in honor of official delegates from affiliated organizations. Voted that the new Board of Directors meet at the close of the Saturday afternoon session of the National Council. President Everett appointed Miss Sanford and Miss Hildebrandt as the committee to draw up fitting resolutions concerning the recent tragic death of Prof. J. W. Young of Dartmouth. Voted that Mr. Rankin's Committee on Handbook in Mathematics be continued another year with no appropriation. The president instructed the secretary to send greetings to Mr. Harry English, a resident of Washington, D.C., and a former member of the Board of Directors, who has been critically ill. Secretary Schreiber asked Mr. Wallis, chairman of the local committee, together with Mr. Myers also of the local committee, to personally take this greeting to Mr. English together with an appropriate floral tribute. The meeting adjourned.

Annual Business Meeting, February 20, 9:15 a.m.
Lecture Room, Bureau of Standards

President Everett called the meeting to order and asked for the treasurer's report which was presented by Mr. Schreiber and accepted as read. Secretary Schreiber was asked to read the minutes of the previous annual meeting held in Detroit, but owing to the lack of time it was voted that the minutes be approved as printed in the May issue, 1931, of THE MATHEMATICS TEACHER. M. J. B. Orleans of New York City made the following motion: "In view of the note touched upon in their references to the importance of the problem of individual differences in mathematics by both speakers at the Friday evening meeting, and in view of the mention of the same problem before the close of the Detroit meeting in 1931, and in view of the feeling among teachers all over the country that one of the obstacles to good work in mathematics is our failure to solve this problem, it is moved that the National Council of Teachers of Mathematics appoint a committee to conduct a thorough study of the question of individual differences in mathematics and to present its first report at the annual meeting in 1933." Carried. Mr. H. C. Barber amended the above motion by referring action in the matter to the Board of Directors. The amendment carried and the motion with amendment carried. Mr. Ralph Beatley of Harvard University made the following motion: "That the National Council of Teachers of Mathematics reaffirm its interest in the question of geometry as expressed at the annual meeting in Detroit, and hope that the Board of Directors will find it possible to initiate and carry forward a study of the whole question of geometry in our schools." Carried. Mr. Walter S. Pope, Cicero, Illinois, moved that the National Council employ a court stenographer to take down the complete transactions of the annual meetings. The motion was lost. Mr. C. M. Austin, Oak Park, Illinois, moved that the National Council through its Board of Directors establish a Committee on Resolutions. Carried. Secretary Schreiber as chairman of the Ballot Committee presented the results of the annual election:

For President, 1932-34: William Betz, Specialist in Mathematics, Rochester, New York—135 votes; J. O. Hassler, University of Oklahoma, Norman, Oklahoma—64 votes.

For Second Vice-President, 1932-34: E. B. Lytle, University of Illinois, Urbana, Illinois—88 votes; Mary Potter, Supervisor of Mathematics, Racine, Wisconsin—106 votes.

For Members of the Board of Directors, 1932-35: Ralph Beatley, Harvard University, Cambridge, Massachusetts—98 votes; John P. Everett, State Teachers College, Kalamazoo, Michigan—108 votes; Elsie P. Johnson, High School, Oak Park, Illinois—109 votes; Raleigh Schorling, University of Michigan, Ann Arbor, Michigan—127 votes; John C. Stone, Teachers College, Montclair, New Jersey—89 votes; E. H. Taylor, State Teachers College, Charleston, Illinois—62 votes.

The following were then declared elected:

For President: William Betz.

For Second Vice-President: Mary Potter.

For Members of the Board of Directors: Elsie P. Johnson, John P. Everett, Raleigh Schorling.

The meeting then adjourned to visit the Bureau of Standards. This privilege was extended to the members of the Council through the courtesy of Mr. George K. Burgess, Director of the Bureau of Standards.

Directors' Meeting, February 20, 4:15 p.m.
English Room, the Raleigh Hotel

Present: John P. Everett; W. S. Schlauch; Martha Hildebrandt; Edwin W. Schreiber; William D. Reeve; Vera Sanford; C. M. Austin; Harry C. Barber; William Betz; Mary S. Sabin; C. Louis Thiele; Mary A. Potter; Elsie P. Johnson. Absent: Herbert E. Slaught; Marie Gugle, Raleigh Schorling.

President John P. Everett presided at this meeting of the Board of Directors. The first item of business was to fill the vacancy on the Board of Directors caused by the election of Mr. William Betz to the office of president. It was moved by Mr. Austin, seconded by Mr. Reeve, that Miss Mary Kelly of Wichita, Kansas, be appointed to fill the unexpired term of Mr. William Betz. Carried. (The term is for one year, 1932-33.)

It was moved by Mr. Barber, seconded by Mr. Thiele, that the incoming president, Mr. Betz, appoint a Committee on Individual Differences with Mr. J. B. Orleans of New York a member. Carried.

It was moved by Mr. Barber, seconded by Mr. Betz, that the Committee on Individual Differences carry with it a maximum appropriation of fifty dollars. Carried.

It was moved by Mr. Barber, seconded by Mr. Betz, that a Committee on Geometry be appointed with Mr. Ralph Beatley of Har-

vard and Mr. Joseph P. McCormick of New York as members of the committee. Carried.

It was moved by Mr. Barber, seconded by Mr. Betz, that the Geometry Committee carry with it a maximum appropriation of fifty dollars. Carried.

It was moved by Mr. Barber, seconded by Mr. Betz, that the Committee on Co-operation with Examining Boards be continued with Mr. Schlauch as chairman. Carried.

It was moved by Mr. Barber, seconded by Mr. Schlauch, that the appointment of a Committee on Resolutions be left to the discretion of the incoming president, Mr. Betz. Carried.

It was moved by Mr. Barber, seconded by Miss Hildebrandt, that the president and secretary report to the Board of Directors as early as possible the membership of the above named committees. Carried.

The following report of the Committee on Directors' Expenses was presented and accepted:

The Committee is in favor of continuing the rule now in force of paying not more than fifty ($50.00) dollars to any member of the Board except in the case of a member who comes from a long distance; the Committee is in favor of paying the railroad fare of such a member.

(Signed) Mary S. Sabin, C. M. Austin, W. D. Reeve.

Dr. Vera Sanford audited the books of the treasurer and reported to President Betz that his accounts were correct. (See report at the end of these minutes.)

General Meeting, Friday Evening, February 19, 8:00 p.m.
Ballroom of the Raleigh Hotel

President John P. Everett officially opened the Thirteenth Annual Meeting of the National Council of Teachers of Mathematics. Secretary Schreiber explained the method of registration and called attention to the sale of the seventh yearbook on the Teaching of Algebra. Mr. William C. Myers, assistant principal of McKinley High School, Washington, D.C., addressed the audience on "Curriculum Adjustments." The second address of the evening was made by Professor William David Reeve, Teachers College, Columbia University, entitled "A Comparative Study of the Teaching of Mathematics in the United States and Germany." Both of these papers were well received and were published in the May issue of THE MATHEMATICS TEACHER.

Breakfast for Official Delegates from Affiliated Organizations
Saturday, February 20, 7:30 a.m.

There were thirty-two present at this breakfast conference, made up of official delegates and Officers and Directors of the National Council. Mr. Schlauch, First Vice-President, presided and Mr. William Betz was the official spokesman, representing the officers and directors of the National Council. Mr. Betz drew the analogy that the National Council might be represented by the trunk of a tree, the affiliated organizations the branches of the tree, and the individual members all over the United States as the leaves of the tree. The main thesis of his remarks was to the effect that success could be accomplished only through complete co-operation. Mr. Schlauch then asked for suggestions from official delegates. Dr. Elizabeth B. Cowley, of Pittsburgh, Pennsylvania, reported for her "section" and her remarks were very encouraging. Mr. A. Brown Miller from Cleveland, Ohio, reported conditions in the Cleveland area. Mr. Walter S. Pope of Cicero, Illinois, reported for the Chicago Men's Mathematics Club and called to the attention of the group that nine of the officers and directors of the National Council had at one time or another been members of the Chicago Mathematics Clubs. Mrs. Elsie P. Johnson reported for the Women's Mathematics Club of Chicago and vicinity. Mr. Nathan Silberstein reported on the state of affairs in New York City. Miss Ruby Flannery of Denver, Colorado, reported conditions in and about Denver. It was definitely felt by all those present that this type of conference between the officers of the National Council and representatives of affiliated organizations is decidedly worth while and should be an annual affair.

General Meeting, Saturday, February 20, 2:00 p.m.
Ballroom of the Raleigh Hotel

The meeting was opened by President Everett who introduced as the first speaker Mr. Harry C. Barber of Phillips Exeter Academy, a recent ex-president of the National Council, who gave us a very interesting message on "Improving America's Mathematics." The second speaker of the afternoon was Miss Beulah I. Shoesmith of Hyde Park High School, Chicago, Illinois, whose address on "What Do We Owe to the Brighter Pupil" was exceptionally interesting and very well received. First Vice-President W. S. Schlauch was the third speaker on the topic "Regents' and College Entrance Board Examina-

tions in Mathematics." Through a series of charts Professor Schlauch presented distinct evidence on certain trends in official examinations.

Annual Banquet, February 20, 6:30 p.m.
Ballroom, the Raleigh Hotel

The annual banquet of the National Council was attended by one hundred fifty-nine members and guests. President Everett acted in the capacity of toastmaster and introduced the newly elected officers of the Council: Miss Mary A. Potter, Second Vice-President; Mrs. Elsie P. Johnson, Director. Prof. Raleigh Schorling, a new member of our Board of Directors, was absent due to illness. First Vice-President Martha Hildebrandt was presented to the group, as was our retiring vice-president, Prof. W. S. Schlauch. Miss Marie Gugle, a Director of the National Council and a recent ex-president, was presented as the only person present who had attended each of the thirteen annual meetings of the National Council. Mr. C. M. Austin, the first president of the National Council, gave a fitting tribute to the two retiring members of our Board of Directors, Mr. John R. Clark and J. O. Hassler. Mr. Clark has been officially identified with the National Council since its organization in 1920. He was for many years Editor of THE MATHEMATICS TEACHER. Mr. Hassler has been a member of the Board of Directors for four years and has done much to preach the gospel "out where the West begins." Mr. Harry C. Barber in brief stated "The King is dead! Long live the King!" As official representative of the Board of Directors he presented our retiring president, Mr. John P. Everett, with a token of our regard and esteem for his services as the leading officer of the National Council for the past two years. Mr. William J. Wallis, chairman of the Local Committee, expressed his deep appreciation of the action of the Board of Directors in sending Mr. Harry English sincere greetings and a floral tribute. Mr. English has been seriously ill for a long time. He was a member of our Board of Directors for two terms.

During the course of the evening we were favored by several musical selections effectively rendered by a guest singer secured by the Local Committee. Dr. W. D. Reeve was asked to introduce the speaker of the evening, Dr. E. R. Hedrick of the University of California at Los Angeles. Professor Hedrick addressed us on the subject "What Mathematics Means to the World." His address was

a fitting climax to the Thirteenth Annual Meeting. A complete report of this splendid address will appear in THE MATHEMATICS TEACHER. The retiring president, John P. Everett, then called upon the newly elected president, William Betz of Rochester, New York, who briefly presented to the group some of the problems immediately confronting the National Council of Teachers of Mathematics. And so ended the Thirteenth Annual Meeting.

(Signed) EDWIN W. SCHREIBER, *Secretary.*

ATTENDANCE

NOTE: See Key to Code at end of list

California (4)

Los Angeles

Hedrick, Earle R. (W) Univ. of Calif., Los Angeles, California
Helmholz, Yedder Feldman (G) Univ. of Calif., Los Angeles, California
Neeland, Jean (G-1) Univ. of Calif., Los Angeles, California

Pacific Grove

Hoffman, Mabel (W) Pacific Grove, California

Colorado (2)

Denver

Flannery, Ruby (W) East High School, Denver, Colorado, Official Delegate of C.O.L. Club
Sabin, Mary (R-5) Denver, Colorado, Official Delegate of Denver Club

Connecticut (1)

Hartford

Wheeler, Dorothy S. (W) Bulkeley High School, Hartford, Connecticut

Delaware (1)

Newark

Gallaher, Anna E. (G-1) Newark High School, Newark, Delaware

District of Columbia (61)

Albert, Gertrude (R-2) Central High School, Washington, D.C.
Alwine, D. C. (W) Stuart Jr. High School, Washington, D.C.
Amig, Margaret (W) Central High School, Washington, D.C.
Atwood, Nancy (G-1) Shaw Jr. High School, Washington, D.C.
Beller, Sadie W (W) Stuart Jr. High School, Washington, D.C.
Bergin, Katherine (G-1) Tyler School, Washington, D.C.
Bianchi, Marian E. (G-1) Randall Jr. High School, Washington, D.C.
Birtwell, Bertha (R-2) McKinley High School, Washington, D.C.
Bogan, Gertrude (W) Deal Jr. High School, Washington, D.C.
Boyd, Norma E. (R-2) Garnet Patterson Jr. High School, Washington, D.C.
Bragg, Frances M. (W) Stuart Jr. High School, Washington, D.C.
Calloway, Caroline (R-3) Dunbar High School, Washington, D.C.
Chaffee, Arthur (G-2) D. C. Heath & Co., Washington, D.C.
Clark, Chester (W) McKinley School, Washington, D.C.
Coombs, Daniel (R-2) Central High School, Washington, D.C.

Crowder, Glenna (W) Wilson Teachers College, Washington, D.C.
Ebaugh, Harriet E. (R-2) McKinley High School, Washington, D.C.
Edmonton, J. B. (R-2) Western High School, Washington, D.C.
Evans, Harriet (R-2) Langley Jr. High School, Washington, D.C.
Eymard, Sr. W. (G-1) Sacred Heart High School, Washington, D.C.
Gilbert, Lee E. (R-2) Central High School, Washington, D.C.
Grubbs, Ethel Harris (R-4) Armstrong High School, Washington, D.C.
Gumey, Harry (G-1) American University, Washington, D.C.
Hammond, Ida (R-2) McKinley High School, Washington, D.C.
Hammond, Jennie (G-1), Washington, D.C.
Harrington, Katherine (W) Western High School, Washington, D.C.
Himbal, Anna A. (W) Hine Jr. High School, Washington, D.C.
Holbrook, Edna (G-1) Central High School, Washington, D.C.
Johnson, Georgie S. (G-1) Shaw Jr. High School, Washington, D.C.
Kellerman, Gertrude (R-2) Central High School, Washington, D.C.
Kupfer, Julie A. (R-2) Eliot Jr. High School, Washington, D.C.
Lloyd, Oliver (W) Mount Vernon Seminary, Washington, D.C.
McGrath, C. M. (R-2) Eastern High School, Washington, D.C.
Minich, William G. (W) McKinley High School, Washington, D.C.
Mitchell, Herbert F. (R-2) McKinley High School, Washington, D.C.
Newton, Beverly (W) Wilson Teachers College, Washington, D.C.
Peters, Gladys L. (W) Garnet Patterson Jr. High School, Washington, D.C.
Philpitt, Blanche (G-1) Cranch, Washington, D.C.
Philpitt, Helen (G-1) Wilson Teachers College, Washington, D.C.
Price, Adele (G-1) Hine Jr. High School, Washington, D.C.
Randolph, M. E. (G-1) Shaw Jr. High School, Washington, D.C.
Richmond, Susan (W) Western High School, Washington, D.C.
Ross, Gro. A. (R-2) Central High School, Washington, D.C.
Rush, Andrew (W) Redempterist House, Washington, D.C.
Scheier, Maurice A. (W) Holy Name College, Washington, D.C.
Shafer, Eloise (W) Western High School, Washington, D.C.
Shelp, Germide A. (R-2) Eastern High School, Washington, D.C.
Simon, Isaac (G-1) Eastern High School, Washington, D.C.
Sinder, Mrs. Alice D. B. (W) Alice Deal Jr. High School, Washington, D.C.
Snoddy, Margaret L. (W) Washington, D.C.
Staples, Helen D. (W) Eastern High School, Washington, D.C.
Stevens, Lucille L. (W) Business High School, Washington, D.C.
Strickler, Grace (G-1) MacFarland Jr. High School, Washington, D.C.
Taylor, James H. (W) The George Washington Univ., Washington, D.C.
Tennyson, J. A. (W) Langley Jr. High School, Washington, D.C.
Thorn, Lulu B. (W) Hine Jr. High School, Washington, D.C.
Wallis, William (R-3) Public Schools, Washington, D.C.
Weider, Frank M. (W) George Washington Univ., Washington, D.C.
Wilder, Jennie (R-5) Dunbar High School, Washington, D.C.
Wilson, Elizabeth (G-1) Washington, D.C.
Worthley, Mary G. (W) McKinley High School, Washington, D.C.

Illinois (15)

Chicago

Johnson, J. T. (R-6) Chicago Normal College, Chicago. Official Delegate of Chicago Men's Mathematics Club

Shoesmith, Beulah I. (R-3) 1404 E. 56th St., Chicago, Illinois

Stone, Charles A. (B-7) University of Chicago, Chicago, Illinois

Cicero

McDonald, J. Russell (R-2) Morton School, Cicero, Illinois. Official delegate of Chicago Mathematics Club

McDonald, Mrs. J. R. (G-1) Morton School, Cicero, Illinois

Pope, Martha H. (G-1) J. Sterling Morton School, Cicero, Illinois

Pope, Walter S. (R-5) J. Sterling Morton School, Cicero, Illinois

Snead, E. J. (W-1) F.U.M.A., Cicero, Illinois

Evanston

Wilson, Mildred (R-5) Nichols Intermediate School, Evanston, Illinois

Macomb

Schreiber, Edwin W. (B-9) State Teachers College, Macomb, Illinois

Maywood

Hildebrandt, Martha (R-5) Proviso Twp. High School, Maywood, Illinois

Oak Park

Austin, C. M. (B-12) High School, Oak Park, Illinois

Johnson, Elsie P. (R-5) Oak Park High School, Oak Park, Illinois

River Forest

Luciola, Sister Mary (G-1) Rosary College, River Forest, Illinois

Waukegan

Dady, Margaret (R-2) Township High School, Waukegan, Illinois

Indiana (5)

Muncie

Whitcraft, L. Harper (R-2) Ball State Teachers College, Muncie, Indiana

Terre Haute

Kennedy, Katheryn M. (R-5) Indiana State Teachers College, Terre Haute, Indiana

Miller, Helen B. (G-1) Indiana State Teachers College, Terre Haute, Indiana

Shriner, Walter O. (R-3) Indiana State Teachers College, Terre Haute, Indiana

Shriner, Mrs. W. O. (G-1) Indiana State Teachers College, Terre Haute, Indiana

Kentucky (1)

Louisville

Gregory, M. Cottell (W) Math. Curriculum Com., Louisville, Kentucky

Maryland (12)

Baltimore

Carroll, Louise E. (W) Southern High School, Baltimore, Maryland

Dwyer, M. Ellen (W) Clifton Park School, Baltimore, Maryland

Herbert, Agnes (R-3) Clifton Park School, Baltimore, Maryland

Holloway, Mary E. (W) Roland Park School No. 233, Baltimore, Maryland

McPherson, Eva (W) No. 91, Baltimore, Maryland

Price, Edna T. (W) Clifton Park Jr. High School, Baltimore, Maryland
Roche, Nanette (W) Administration Bldg., Baltimore, Maryland.
Schwatha, John H. (G-1) Southern High School, Baltimore, Maryland
Tarnes, Katherine (W) Hamilton Jr. High School, Baltimore, Maryland

Chestertown
Snodgrass, F. T. (G-1) Washington College, Chestertown, Maryland

College Park
Kalmbach, Virginia (W) Univ. of Maryland, College Park, Maryland

Frederich
Martz, Grace (W) Frederich, Maryland.

Massachusetts (2)

Boston
Stevens, E. M. (G-1) Ginn & Co., Boston, Massachusetts

Cambridge
Beatley, Ralph (R-4) Harvard University, Cambridge, Massachusetts

Michigan (6)

Ann Arbor
Lindell, Selma A. (R-6) University High School, Ann Arbor, Michigan

Detroit
Hobbs, Adelia (R-3) Coudon School, Detroit, Michigan
Thiele, C. L. (R-6) Supv. of Dept., Detroit, Michigan
Worden, Orpha E. (B-12) Teachers College of Detroit, Michigan. Official Delegate of Detroit Mathematics Club

Flint
Loss, Nellie (R-2) Flint, Michigan

Kalamazoo
Everett, John P. (B-9) Western State Teachers College, Kalamazoo, Michigan

Minnesota (2)

Minneapolis
Brueckner, L. J. (W) Univ. of Minnesota, Minneapolis, Minnesota

St. Cloud
Bemis, C. O. (W) Teachers College, St. Cloud, Minnesota

Nebraska (1)

Lincoln
Congdon, Allan R. (W) University of Nebraska, Lincoln, Nebraska

New Hampshire (1)

Exeter
Barber, Harry C. (R-5) Exeter Academy, Exeter, New Hampshire

New Jersey (6)

Elizabeth
Loughren, Amonda (R-2) Jefferson High School, Elizabeth, New Jersey

Newark
Conklin, R. P. (W) Central High School, Newark, New Jersey
Fehr, Howard F. (W) South Side High School, Newark, New Jersey
Stauter, Anna M. (G-1) Newark High School, Newark, New Jersey

Paterson
Rea, Frederick H. (G-1) Non-teaching, Paterson, New Jersey

Westfield

Morton, Josephine (G-1) Roosevelt Jr. High School, Westfield, New Jersey

New York (31)

Buffalo

Crofts, Mary E. (R-4) Fosdick-Masten Park School, Buffalo, New York. Official delegate of Buffalo Club

Podmele, Theresa L. (W) East High School, Buffalo, New York

Poole, Hallie S. (R-4) Lafayette High School, Buffalo, New York. Official delegate of Buffalo Club

Walsh, Sara C. (R-3) East High School, Buffalo, New York. Official delegate of Buffalo Club

Cortland

Sueltz, Ben A. (W) State Normal School, Cortland, New York

Harrison

Arning, Alexander (W) Harrison High School, Harrison, New York

Sickman, Harland (G-1) Harrison High School, Harrison, New York

New Paltz

Lane, Florence A. (R-2) State Normal, New Paltz, New York

New Rochelle

Englebrekt, A. (W) I. E. Young Jr. High School, New Rochelle, New York

New York

Carter, Olive J. (W) The Macmillan Co., New York, New York

Clark, John R. (B-11) Lincoln School, New York, New York

de Booy, Margaret (W) Columbia University Student, New York, New York

Fawcett, H. P. (W) Columbia University, New York, New York

Isham, Franklin (R-2) Stony Brook School, Long Island, New York

Jablonower, Joseph (R-3) Fieldston School, New York, New York

Laverty, Catherine C. (W) P.S. 82, New York, New York.

McCormack, Joseph P. (R-4) Theodore Roosevelt High School, New York New York. Official delegate of New York Experimental Soc.

McLaughlin, Isabel (W) Bryant High School, New York, New York

Orleans, Joseph B. (R-3) George Washington High School, New York, New York. President of Association of Chairmen of Mathematics Departments

Palmer, Maritte (R-2) 64 Manhattan, New York, New York. Official delegate of N.Y.A. of M.T.

Pieper, Joseph H. (R-4) New York University, New York, New York

Quinn, Sister M. Gertrude (W) Our Lady of Mercy Boys High School, New York, New York

Reeve, W. D. (B-11) Teachers College, New York, New York

Schlauch, W. S. (R-4) New York University, New York, New York

Shuman, Rochel (W) Student, Teachers College, Columbia University, New York, New York

Silberstein, Nathan (W) James Monroe High School, New York, New York. Official delegate of Math. Chairmen

Welch, Alberta M. (W) Bryant High School, New York, New York

Niagara Falls

Daw, Elizabeth (W) Gaskill Jr. High School, Niagara Falls, New York

Rochester

Betz, William (B-8) Alexander Hamilton School, Rochester, New York

Thornton, Maude A. (W) Andrews Schools, Rochester, New York

Scarborough

Mattern, R. B. (R-2) Scarborough School, Scarborough, New York

North Carolina (2)

Clinton

Daniel, S. F. (W) Clinton High School, Clinton, North Carolina

Greenville

ReBarker, Herbert (R-3) East Carolina Teachers College, Greenville, North Carolina

Ohio (11)

Athens

Benz, H. E. (R-4) Ohio University, Athens, Ohio

Pickett, Hale (R-2) Athens High School, Athens, Ohio

Bowling Green

Overman, J. R. (B-7) Bowling Green State College, Bowling Green, Ohio

Cincinnati

Becker, Marie (R-3) Walnut Hills High School, Cincinnati, Ohio

Struke, Norma (R-3) Rothenberg Jr. High School, Cincinnati, Ohio

Cleveland

Miller, A. Brown (R-6) Fairmont Jr. High School, Cleveland, Ohio. Official delegate of Cleveland Club

Sanford, Vera (R-6) Western Reserve University, Cleveland, Ohio

Columbus

Gugle, Marie (B-13) Assistant Superintendent of Schools, Columbus, Ohio

Hammond, Rose L. (G-4) Columbus, Ohio

Moore, Charles T. (R-2) South High School, Columbus, Ohio

Oxford

Christofferson, H. C. (R-4) Miami University, Oxford, Ohio

Oklahoma (1)

Norman

Hassler, J. O. (R-5) University of Oklahoma, Norman, Oklahoma

Pennsylvania (8)

California

Salisbury, E. G. (R-3) Teachers College, California, Pennsylvania

Collegeville

Fritsch, Mabel (W) Collegeville School, Collegeville, Pennsylvania

East Stroudsburg

Burrus, Marvin E. (W) State Teachers College, East Stroudsburg, Pennsylvania

Gettysburg

Baker, Ira Y. (R-2) Gettysburg High School, Gettysburg, Pennsylvania

Leferer, G. W. (W) High School, Gettysburg, Pennsylvania

Narberth

Fritich, Mary (G-1) Narberth, Pennsylvania

Philadelphia

Rorer, J. T. (R-2) William Penn High School, Philadelphia, Pennsylvania

Pittsburgh

Cowley, Dr. Elizabeth B. (R-3) Allegheny Senior High School, Pittsburgh, Pennsylvania. Official delegate of Math. Section of Pennsylvania State Ed. Association

Rhode Island (3)

Providence

Cooper. L. Louise (R-3) Technical High School, Providence, Rhode Island

Hildreth, Alice F. (R-3) Technical High School, Providence, Rhode Island

Talley, Margaret (W) Wheeler School, Providence, Rhode Island

South Carolina (1)

Greenville

Short, Vivian (W) Greenville Woman's College, Greenville, South Carolina

Virginia (17)

Alexandria

Dickert, Eddie (G-1) Alexandria High School, Alexandria, Virginia

Hill, Ida (W) Alexandria High School, Alexandria, Virginia

Rowlett, Louise (W) Alexandria High School, Alexandria, Virginia

Shackelford, Grigsby (G-1) Episcopal High School, Alexandria, Virginia

Charlottesville

Wingfield, R. C. (G-1) University of Virginia, Charlottesville, Virginia

Fairfax

Woodson, W. T. (G-1) Fairfax, Virginia

Farmville

Taliaferro, Carrie (W) State Teachers College, Farmville, Virginia

Fredericksburg

Mills, Helen (W) State Teachers College, Fredericksburg, Virginia

Glen Allen

Allport, Mrs. G. D. (W) Glen Allen High School, Glen Allen, Virginia

Hampton

Atkinson, Robert (W) Hampton Institute, Hampton, Virginia

Perkins, Herbert K. (W) Hampton Institute, Hampton, Virginia

Harrisonburg

Harnsberger, Grace (G-3) Harrisonburg, Virginia

Richmond

Givens, C. W. (W) John Marshall High School, Richmond, Virginia

Mottley, Bessie M. (G-1) Richmond, Virginia

Wilson, Ruth (W) Thomas Jefferson High School, Richmond, Virginia

Woodberry Forest

Lord, W. K. (R1) Woodberry Forest High School, Woodberry Forest, Virginia.

Shackelford, A. C. (G-1) Woodberry Forest High School, Woodberry Forest, Virginia

West Virginia (3)

Bluefield

Akers, Bernice (W) Ramsey Jr. High School, Bluefield, West Virginia

Jones, Connor (W) Beaver High School, Bluefield, West Virginia

Parkersburg

Morgan, Agnes (R-2) Parkersburg High School, Parkersburg, West Virginia

Wisconsin (1)

Racine

Potter, Mary A. (B-10) Washington Park High School, Racine, Wisconsin

Attendance by States

	G	*W*	*R*	*B*	*T*
1. California	2	2	0	0	4
2. Colorado	0	1	1	0	2
3. Connecticut	0	1	0	0	1
4. Delaware	1	0	0	0	1
5. District of Columbia	16	26	19	0	61
6. Illinois	3	1	8	3	15
7. Indiana	2	0	3	0	5
8. Kentucky	0	1	0	0	1
9. Maryland	2	9	1	0	12
10. Massachusetts	1	0	1	0	2
11. Michigan	0	0	4	2	6
12. Minnesota	0	2	0	0	2
13. Nebraska	0	1	0	0	1
14. New Hampshire	0	0	1	0	1
15. New Jersey	3	2	1	0	6
16. New York	1	15	12	3	31
17. North Carolina	0	1	1	0	2
18. Ohio	1	0	8	2	11
19. Oklahoma	0	0	1	0	1
20. Pennsylvania	1	3	4	0	8
21. Rhode Island	0	1	2	0	3
22. South Carolina	0	1	0	0	1
23. Virginia	7	9	1	0	17
24. West Virginia	0	2	1	0	3
25. Wisconsin	0	0	0	1	1
	40	78	69	11	198

Note: G = green registration card (visitor or guest)

W = white registration card (member attending his first annual meeting)

R = red registration card (member attending 2-6 annual meetings)

B = blue registration card (member attending ½ or more annual meetings)

In the preceding list the number following the letter indicates the number of annual meetings attended, thus R-3 means 3 meetings attended.

Report of the Secretary-Treasurer as Treasurer, February 15, 1932

Receipts

Balance Feb. 1, 1931 .... $1856.40

W. D. Reeve, Mathematics Teacher $1357.02

W. D. Reeve, Year books		450.01
W. D. Reeve, Year books		385.00
		2192.03
Other Assets:		
New York Telephone Bond	$ 982.50	
Savings	302.02	
Certificate of Deposit	450.01	
Total		$1734.53
		$4048.43

EXPENDITURES

Annual Meeting:		
Director's Expenses	$477.32	
Program Speakers	158.76	
Local Committee	16.50	
Printing	27.00	
Official Delegate Luncheon	47.00	
Guest Banquet Tickets	20.00	
		$ 746.58
President's Office:		
Traveling Expenses, Telephone, Stamps, etc		47.13
Editor's Office:		
Yearbook		7.60
Geometry Committee		23.90
Standing Committee on Examinations		102.23
Committee on Extension of the Council		123.15
Secretary-Treasurer's Office:		
Certificate of Deposit	$450.01	
Supplies, Stationery, etc	155.80	
Secretarial Service	500.00	
MATHEMATICS News Letter	24.50	
Annual Dues to European Mathematics Societies	16.37	
Recording Fee	1.30	
		1147.98
Total Expenditures		$2198.57
Cash on Hand		1849.86
		$4048.43

(Signed) EDWIN W. SCHREIBER, *Treasurer*

The Problem of Individual Differences in Mathematics Classes[1]

By Mabel Sykes
Bowen High School, Chicago, Illinois

This study was undertaken in connection with sabbatical leave from one of the Chicago high schools and covers over fifty schools, twenty-five of which were visited by the writer. The facts concerning the others were obtained from replies to letters. The following general methods of dealing with individual differences are discussed:

1. Ability grouping.
2. Differentiated assignments.
3. Individual instruction.
4. Special help for slow or failing pupils or for repeaters.

Ability Grouping

Ability grouping seems to be the most general method of dealing with individual differences. Two phases of the subject are of importance: the teaching problem in classes formed of different ability levels and administrative considerations.

A. THE TEACHING PROBLEM

In general this is left with the class teacher. Usually the same book is used but the slower class omits some of the work. Pupils in such classes receive more explanations and thus have a better chance to get difficulties discussed than if they were in heterogeneous groups. The success of the plan as thus used is variously reported. Some schools think that it is worth while largely for the sake of the accelerated pupils; others think that the teacher gets more help from it than the pupils. There is no doubt but that the success in the case of retarded pupils depends upon the sympathetic attitude of the teacher and her independence and resourcefulness in planning work. Unless such teachers can be obtained the work for the slower classes

[1] Readers of The Mathematics Teacher will be interested in a short article by Dr. Arthur S. Otis on "Ability Grouping" in *School and Society* for July 23, 1932.

must be especially provided for in some way or the success of the plan for the slower pupils may be questioned.

In one school in which classes of two ability levels were used, the members of the mathematics department met and carefully planned the work; for example, the geometry was divided into units and the theorems and exercises to be required of each group were carefully laid out in parallel columns. In this way the work to be omitted by the slower group was uniform throughout the school and at once evident from the syllabus so prepared. A similar syllabus was prepared for algebra classes.

In some schools an entirely different book is used for classes composed of pupils of the lowest ability level. There are not many texts on the market suitable for this work. It is hoped that more are in preparation.

Occasionally when it is possible to program from three to six classes at the same period, shifts are made from one class to another. In one such case the lowest group in ninth grade algebra was given a modified form of arithmetic. Occasionally in slower groups a few minutes' daily drill in arithmetic is found desirable.

Sometimes three term and two term classes in the same subject are arranged. In such cases the two groups cover the same ground but the slower classes take more time for the work. This is based on the theory that the difference in pupils' grasp of the subject is largely a matter of speed. In one case it was reported that only about one quarter of the pupils could do ninth grade algebra in two terms. One wonders if the work as planned was not in general too hard.

One case was reported in which slow pupils were assigned to double period classes.

The above gives some idea of the attempts that are being made to adapt the work to retarded pupils.

B. ADMINISTRATIVE CONSIDERATIONS

When the work done in classes composed of pupils from the lowest ability level is essentially different from and easier than that done in other classes, that work cannot be used for college entrance or, in New York State, for admission to the Regents' Examinations. In such cases a pupil must take an extra semester in the regular classes before he can use his mathematics for such purposes. How this matter is managed on office records and in counting credits for graduation is a subject that needs further investigation.

The basis on which the division into groups may be made has been ably discussed by Mr. Ferdinand Kertes in THE MATHEMATICS TEACHER for January, 1932. The reader is referred to this article for details.

When the tests on which the segregation is based are given in the high school after the semester is begun, serious program difficulties are liable to result unless the school is large or a small number of courses are offered. This difficulty is met in two ways:

1. Two or more classes in the same subject are programmed for the same period. After the semester has begun the necessary tests are given and pupils are shifted from one class to another that meets at the same period. The number of classes that can be so programmed depends upon the size of the school and varies from two to six in the schools studied.

2. Sometimes the selection of pupils for classes of different ability levels is made in the elementary schools before the pupils enter high school. Sometimes a test of algebra ability is given, the papers are marked and sent to the high school. From the results of this test the high school assigns each pupil to what appears to be the proper group or to some course that does not require algebra. Sometimes the elementary teachers fill out questionnaires sent out by the high school. Such questionnaires usually include the recommendation of the eighth grade teacher for the regular classes and for opportunity classes. This plan is said to work well in schools where it is used.

Differentiated Assignments

This includes cases in which the assignment for pupils in the same class is made to fit different levels of ability. It goes by various names, such as, differentiated assignments, flexible assignments or contract work. The assignment so made may be given to cover work for one or more days or for an entire unit of work, and may be used with or without a text. Sometimes more than one text is referred to in the assignment.

Differentiated assignments have certain advantages. They can be used where ability grouping is not feasible. Slow pupils profit by contact with the brighter ones while the brighter ones use just as much initiative and do just as much independent thinking as in ability grouping. To insure success the class must be under the guidance of a teacher especially sympathetic and resourceful. The observer

wondered that she did not encounter more of this work as there are practically no administrative difficulties connected with it.

The following illustrations of such work were reported:

1. Pupils designated as honor pupils cover an honor syllabus in addition to the regular syllabus. This consists of theorems and exercises not on the regular outline, puzzle problems and papers on the history of mathematics. Such pupils present their exercises and papers for the benefit of the entire class. (Franklin K. Lane High School, New York City. See report in THE MATHEMATICS TEACHER October 1930.)

2. The work is divided into units. The teacher devotes one or more class periods introducing the unit. This introduction includes an oral preview to learn what the class may already know about the unit as well as a presentation of that unit. The assignments, however, seldom cover more than two or three days' work so that the completed unit includes several assignments.

3. The first few weeks of the subject are used for general discussion after which differentiated assignments are given without dividing the subject into units. The subject covered by the assignment is always definitely given. The assignments are short, not covering more than two or three days' work, and are made for A, B, and C contracts. The C contract always covers more than the minimum essentials. The teacher reports that pupils get very enthusiastic and that she often is placed in the embarassing position of either having to kill herself or dampen their ardor.

4. The assignment is like a weekly contract. A minimum is always carefully outlined. One day a week the teacher does most of the talking, sometimes giving a preview, sometimes an organized review. At least one oral recitation a week is required of each pupil. One written test is given each week which is carefully marked; this test may not take the entire period. Written work is accepted; it is sometimes done in class and sometimes not.

Individual Instruction

The South Philadelphia High School for Girls is conducted on the Dalton Plan. (For details see "Educating for Responsibility" by Dr. Lucy L. W. Wilson; Macmillan 1926.)

Special Help for Slow or Failing Pupils or for Repeaters

A very large percentage of the schools report special help given

to pupils who need it, either in class or in conference periods. The crowded condition of the programs of both teachers and pupils, however, limits the possibility of such work.

One city conducts classes on Saturday mornings for pupils who need help on class work or who have work to make up. Attendance is voluntary.

One school reports lesson sheet classes conducted for pupils who, for one reason or another, are behind the regular classes. In this school only one year of mathematics is required for graduation so that pupils who are unable to do the regular work in algebra are assigned to such classes for special arithmetic work. Each pupil works at his own rate of speed. When a lesson is completed the pupil takes it to the teacher for correcting and recording, and, if it is satisfactory, receives the next lesson. Pupils who make up work in this way are allowed to go into the regular classes when able to do so.

In some schools pupils who failed are grouped in special repeater classes. Both repeaters and beginners would seem to profit by such an arrangement.

Miscellaneous Considerations

The unit or topic method is used more or less informally in many schools, probably because it affords the best opportunity to emphasize important methods or theorems and to show the relation of one theorem or method to others. It does not of necessity, however, offer any particular help to weak pupils.

One school reports the use of weighted credits:

A mark of 60-69 gives 1 credit.	A mark of 85-89 gives 4 credits.
A mark of 70-79 gives 2 credits.	A mark of 90-94 gives 5 credits.
A mark of 80-84 gives 3 credits.	A mark of 95 or above gives 6 credits.

170 credits are required for graduation. While 80 is the passing mark a little credit towards graduation is allowed if the mark is between 60 and 79.

The situation in regard to third semester, or intermediate, algebra is very unsatisfactory in many cases and needs further study. One school recently gave a test in first year algebra to 600 pupils taking second semester geometry. This test consisted of 25 extremely easy exercises; these exercises were short and could be marked either "right" or "wrong." The median was 40. 154 pupils made as much as 50 per cent. The writer wonders if this is indicative of the results

obtained from all of our ninth grade algebra teaching and what is the best way to adapt our third semester algebra to such conditions.

Many phases of the subject need further study, The observer did not feel that she had any right to inquire into the percentages of failures. Such an inquiry might indicate the real success, or failure, of the various plans used. Moreover, further study might bring to light other variations of the general plans reported. It did not seem wise to report textbooks and tests used until a longer list could be given. As stated at the beginning of this article, this study was purely a private matter.

Sir Christopher Wren*

Born October 20, 1632

Died February 25, 1723

CHRISTOPHER WREN'S FAME as an architect is so great that we are apt to forget that he was Savilian Professor of astronomy at Oxford for a period of twelve years, and that he engaged in several pieces of research in mathematics, studying the properties of the cycloid and of the hyperboloid of revolution. He was also one of the founders of the Royal Society and served as its president for several years.

His career as an architect seems to have begun somewhat by accident. In 1663 he was appointed one of the commissioners charged with planning repairs for St. Paul's Cathedral in London. This building was demolished in the Great Fire of 1666, and Wren was appointed architect for the new building. Beside St. Paul's, Wren designed many other churches in and about London. He is also responsible for the Greenwich Observatory, Chelsea Hospital, buildings at Oxford and Cambridge, and for a part of Hampton Court.

His drawing instruments in the Old Ashmolean Museum at Oxford remind one of his career in that University while the dome of St. Paul's makes one think of the architect whose tomb has the inscription "Si momumentum requiris, circumspice."

* See Frontispiece.

NEWS NOTES

THE SEVENTH MEETING of the Men's Mathematics Club of Chicago and Metropolitan Area was held on April 15, 1932, at the Central Y.M.C.A. The following program was given:

1. Report on the Washington meeting by Messrs. Austin, Pope and McDonald.
2. "Teaching Skill in Quantitative Thinking through Concrete Problems in Ninth Grade Algebra," by W. V. Strawe, Harrison Technical High School.
3. "Mathematics in Development Work," by Mr. K. L. Scott, Western Electric Company.

The 1932 Annual Meeting of the Mathematics Section of the Westchester County Teachers Association (N.Y.) was held on Saturday, April 23, at the White Plains High School. The following program was rendered:

"Experimentation in the Teaching of Mathematics in a High School," by Joseph B. Orleans, George Washington High School, New York City, Member of State Mathematics Committee.

Discussion on the revised Intermediate Algebra syllabus and third year mathematics syllabus. Mr. Orleans has kindly consented to answer any questions.

Leaders: Adelaide LeCount, New Rochelle; Concetta Testa, Montrose; Robert O. Black, White Plains; Constance Hahn, Tarrytown; Dorothy Keeler, Port Chester.

Miss Ruth Barry presided at the meeting.

At a recent conference on the teaching of mathematics in secondary schools Mr. Elmer R. Bowker of the Boston Public Latin School gave a detailed account of parallel instruction in algebra and geometry in Grades 9, 10, 11, as practised in his school. The work of Grade 9 is 90 per cent algebra and 10 per cent geometry; Grade 10 is 50 per cent algebra and 50 per cent geometry; and Grade 11 is 60 per cent algebra and 40 per cent geometry. It will be observed that in the three years taken together exactly two thirds of the work is in algebra and one third in geometry. This accords with the recent tradition of two units of algebra and one unit of geometry. Mr. Bowker admitted that the details of the plan were by no means settled, though his department seems to be committed, for the present at least, to the general idea.

Marks are given in "mathematics," never in algebra or geometry separately. This does not mean that the two subjects are fused in the classroom. They are not. This mode of marking makes it possible to promote a student who is deficient in one subject if his work in the other subject is good enough to balance this deficiency. On the other hand, the only way to get credit for two units of algebra and one unit of geometry is to do satisfactory work in all three years. This makes for continuous and sustained effort.

By means of parallel instruction in algebra and geometry both subjects are kept continually before the pupils for three years. This procedure avoids the great loss of time and effort experi-

enced in most schools in Grade 11, when the study of algebra is resumed after a lapse of fourteen months. It is expected that those who plan to enter college will take the Comprehensive Examination in elementary mathematics. They are ready for this at the end of the eleventh grade. Those who wish to enter college under the New Plan and desire to offer mathematics as one of the four subjects in which they will be examined must take advanced mathematics in Grade 12 or else review for the Comprehensive Examination in elementary mathematics on their own initiative. Discussion of this point showed that many schools in this vicinity make definite provision to this end by allowing students to count for credit a special review course in elementary mathematics devised for this very purpose and given in Grade 12.

Mr. C. H. Mergendahl of the Newton High School recognized in parallel instruction a cure for the present evil of taking time belonging to algebra and using it to support the geometry, which is insufficiently mastered in the tenth grade. He suggested that schools that devote the bulk of Grade 9 to algebra could give three periods a week in Grade 10 to algebra, and two periods to geometry, reversing these figures in Grade 11. Other schemes of dividing the time were suggested. This substitution of continued small doses in place of larger doses, with larger intervals for forgetting, raises an interesting and important question concerning the economy of learning, as was pointed out by Mr. Taylor, also of Newton.

The Spring meeting of the Mathematics Teachers of the Bay Section of the California State Teachers' Association was held at the Hotel Leamington in Oakland, California, on April 23, 1932. The following addresses were given:

1. "The Theory of Relativity," by Professor Victor F. Lenzen, Department of Physics, University of California.

2. "The Unit Plan," by James W. Hoge, Supervisor of Mathematics at the University of California High School.

The mid-winter meeting of the Association of Teachers of Mathematics in New England was held at the Portland (Me.) High School on Saturday, March 12, 1932. The program was as follows:

Forenoon Session

"Time Saving Devices in Geometry," by Miss Ada Bell Kennan.

"A Solar Eclipse," by Professor Harry R. Willard.

Afternoon Session

"Pitfalls in Algebra," by W. L. Vosburgh, Boston Teachers College.

"Check and Double Check," by Professor Titus E. Mergendahl, Massachusetts Institute of Technology.

The forty-fifth meeting of the Association of Mathematics Teachers of New Jersey was held at the State University at New Brunswick on May 7, 1932, with Mr. Roscoe Conkling of the Central High School of Newark, N.J., as chairman. The following program was given:

Presidential Address, "The Mathematics of the Twelfth Year," by Mr. Roscoe P. Conkling, Central High School, Newark, New Jersey.

"A Course in Ninth Grade Mathematics for Pupils Who Are Not Going to College," by Prof. Virgil S. Mallory, Montclair State Teachers College.

"A Report of New Jersey Questionnaire on Ninth Grade Mathematics

With Suggestions," by Miss Amanda Loughren, Supervisor of Mathematics, Elizabeth, New Jersey.

Prof. Virgil S. Mallory of the Montclair Teachers College is the new chairman.

Section A of the A.A.A.S. met on Tuesday, June 21, 1932, at Syracuse University. The following addresses were given:

"Homeomorphic Geometry of the Projective Plane," by Prof. H. M. Gehman, University of Buffalo.

"Logical Foundations for Groups and Fields," by Prof. W. A. Hurwitz, Cornell University.

"An Interpolation," by Prof. J. Shohat, University of Pennsylvania.

The next meeting of the National Council of Teachers of Mathematics will be held at the Hotel Nicollet in Minneapolis on Friday and Saturday, February 17 and 18, 1933. Members of the Council who plan to attend the meeting should write at once to the Hotel Nicollet for reservations. Prices of rooms are as follows:

Without bath, single, \$2; double, \$3.50.

With bath, single, \$2.50, \$3, \$3.50, \$4, \$5, and \$6.

With bath, double, \$4, \$4.50, \$5, \$6, \$7, and \$8.

With bath and twin beds, \$5, \$6, \$7, and \$8.

Parlor (bedroom and bath), 1 or 2, \$10 and \$12.

Parlor (2 bedrooms and bath), 3, \$14 and \$16.

Parlor (2 bedrooms and bath), 4, \$16 and \$18.

NEW BOOKS

The Teaching of Elementary Algebra. By C. V. Durell. Pages viii + 136 G. Bell and Sons, Ltd. London, 1931. Price 3s. 6d.

This book deals with the details of classroom procedure in algebra. It is a practical teaching manual intended for teachers with little experience and for use in training student teachers. However, teachers, of extended experience will find it interesting to compare the suggestions given herein with results of their own experience.

This book is written by a teacher of algebra after many years of experience and as a result of a great deal of discussion with other teachers of the same subject. It ought to be of interest to all American teachers of mathematics.

Plane Geometry. By Frank M. Morgan, John A. Foberg, and W. E. Breckenridge. Pages x + 436. Houghton Mifflin Company, 1931. Price \$1.40.

Whenever a new geometry appears one expects to find some new feature to justify the addition of another text to the already long list of recent publications. The purpose of this book is to provide a sequence of material and method of presentation that will facilitate learning.

After a short introduction the traditional five books are given. The ma-

terial is organized about "congruent triangles," "parallel lines," "quadrilaterals," "circles," "similar triangles," and the like.

To further the emphasis upon understanding rather than memory, a brief analysis is given before each theorem. A large number of original problems are provided for drill work. To make clearer the work in construction problems, the principle of moving picture geometry has been introduced.

The Ingenious Dr. Franklin. Selected Scientific Letters of Benjamin Franklin. Edited by Natham Goodman of Germantown, Pennsylvania. Board. Pages xi + 244. University of Pennsylvania Press, 3438 Walnut Street, Philadelphia, Pennsylvania, 1931. Price $3.00.

Few people in America realize how ingenious Benjamin Franklin really was. This book contains his engaging speculative letters some of which have never before been published. They include references to the famous kite and stove, cold air baths, the first balloon ascensions in Paris, magic squares, daylight saving, and other interesting topics.

Instruction in Ninth Grade Mathematics. By C. N. Stokes. Pages ix + 140. Henry Holt and Company.

The study reported in this book attempts to answer two questions:

1. In ninth-grade mathematics, are individual progress and self-instruction more efficient or less efficient than class or group instruction, as shown by controlled experimentation?

2. In ninth-grade mathematics, do pupils who work independently progress more rapidly or less rapidly than those who are taught by the classroom method?

Chapter I sets forth the problem to be solved. The steps followed in treating the problem are:

1. Chapter II deals with the trends in individualizing instruction.

2. Chapter III is concerned with the nature of the present experimental situation and method of attack.

3. Chapter IV gives a description of the techniques.

4. Chapter V contrasts the results of the two techniques.

5. Chapter VI summarizes the findings and presents certain implications and recommendations.

6. Chapter VII presents some of the recognized limitations of this individualized technique.

All teachers of mathematics who are interested in improving ways of teaching individual differences in ability among high school pupils will profit by reading this book.

Geometric Concepts. By Clara H. Mueller. Pages xi + 205. John Wiley and Sons. Price $1.60.

The purpose which the author had in mind in writing this book was to teach pupils who use it the names and characteristics of the common geometric forms. It is the result of actual experimentation in the classroom and is built up entirely around a consideration of the solid figures.

A psychological and intuitive method of approach is used throughout with the thought that this will bring the pupil closer to the geometric truth than the traditional method of demonstration. Hence one will find in this book no formal demonstration.

Those teachers who regret that we live in a world of three dimensions and teach a geometry of Flatland will be glad to see this book.